MATHEMATICS SOLUTION

(CBCGS MAY – 2018 SEM - 4)

BRANCH – COMPUTER ENGINEERING

Q1] A) Find all the basics solutions to the following problem: (5)

Maximise : $\mathbf{z = x_1 + 3x_2 + 3x_3}$

Subject to : $\mathbf{x_1 + 2x_2 + 3x_3 = 4}$

$\mathbf{2x_1 + 3x_2 + 5x_3 = 7}$

$\mathbf{x_1, x_2, x_3 \geq 0}$

SOLUTION :-

Maximise : $z = x_1 + 3x_2 + 3x_3$

Subject to : $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$

$x_1, x_2, x_3 \geq 0$

No of basic solution	Non basic variable	Basic variable	Equation and values of basic variables	Is solution feasible?	Is solution degenerate	Value of Z	Is solution optimal
1	$x_3 = 0$	x_1, x_2	$x_1 + 2x_2 = 4$ $2x_1 + 3x_2 = 7$ $x_1 = 2, x_2 = 1$	Yes	No	5	Yes
2	$x_2 = 0$	x_1, x_3	$x_1 + 3x_3 = 4$ $2x_1 + 5x_3 = 7$ $x_1 = 1, x_3 = 1$	Yes	No	4	No
3	$x_1 = 0$	x_2, x_3	$2x_2 + 3x_3 = 4$ $3x_2 + 5x_3 = 7$ $x_2 = -1, x_3 = 2$	no	no	-	-

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Q1] B) Evaluate $\int_0^{1+2i} z^2 dz$, along the curve $2x^2 = y$ **(5)**

SOLUTION :-

$y = 2x^2$

$dy = 4xdx$

$z = x + iy$

$dz = dx + idy = dx + i4xdx \;=\; (1 + 4xi)dx$ .... (1)

$\int_0^{1+2i}(x + iy)^2 dz \;=\; \int_0^{1+2i} x^2 - y^2 + 2ixydz = \int_0^1 (x^2 - 4x^4 + 2ix.\,2x^2)(1 + i4x)dx$

$\int_0^1 (x^2 - 4x^4 + 4ix^3 + i4x^3 - 16x^5 i - 16x^4)dx \;=\; \int_0^1 (x^2 - 20x^4 + 8ix^3 - 16x^5 i)dx$

$= \left[\frac{x^3}{3} - \frac{20x^5}{5} + 2i - \frac{16x^6 i}{6}\right]_0^1 \;=\; \frac{1}{3} - 4 + 2i - \frac{8}{3}$

ans $: - \; -\frac{11}{3} - \frac{2i}{3}$

Q1] C) A random sample of size 16 from a normal population showed a mean of 103.75cm and sum of squares of derivations from the mean 843.75 cm^2. Can we say that the population has a mean of 108.75? **(5)**

SOLUTION :-

First we calculate sample standard deviation

$s^2 = \frac{\sum(xi - \bar{x})^2}{n} = \frac{843.75}{16} = 52.73$

(i) The null hypothesis $H_0 : \mu = 108.75$
Alternate hypothesis $H_a : \mu \neq 108.75$

(ii) $t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{103.75 - 108.75}{\sqrt{52.73}/\sqrt{15}} = -2.67$
$|t| = 2.67$

(iii) Level of significance ; $\alpha = 0.05$

(iv) t_α for 5% level of significance and degree of freedom
$v = 16 - 1 = 15$ is 2.131

(v) $2.67 > 2.131$

i.e $t_{calc} > t_{obs}$

we cannot say that mean population is 108.75

Q1] D) If $A = \begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$; Find : sinA **(5)**

$A = \begin{bmatrix} \pi/2 & \pi \\ 0 & 3\pi/2 \end{bmatrix}$; TF : sinA

The characteristics equation is;

$$\begin{bmatrix} \frac{\pi}{2} - \lambda & \pi \\ 0 & \frac{3\pi}{2} - \lambda \end{bmatrix} = 0$$

$$\left(\frac{\pi}{2} - \lambda\right)\left(\frac{3\pi}{2} - \lambda\right) = 0$$

Let $\varphi(A) = \sin A = \alpha_1 A + \alpha_0 I$

Λ satisfies the above equation we have

$$\sin\lambda = \alpha_1\lambda + \alpha_0$$

Putting $\lambda = \frac{\pi}{2}$ and $\frac{3\pi}{2}$

$$\sin\left(\frac{\pi}{2}\right) = \alpha_1\left(\frac{\pi}{2}\right) + \alpha_0$$

$1 = \alpha_1\left(\frac{\pi}{2}\right) + \alpha_0$ ……………………… (1)

$$\sin\left(\frac{3\pi}{2}\right) = \alpha_1\left(\frac{3\pi}{2}\right) + \alpha_0$$

$-1 = \alpha_1\left(\frac{3\pi}{2}\right) + \alpha_0$ ……………………… (2)

Equation (1) – (2);

$$2 = \alpha_1\left(\frac{\pi}{2} - \frac{3\pi}{2}\right)$$

$2 = \alpha_1(-\pi) \qquad = \qquad \alpha_1 = -\frac{2}{\pi}$

And $\alpha_0 = 2$

$$\sin A = -\frac{2}{\pi}\begin{bmatrix} \frac{\pi}{2} & \pi \\ 0 & \frac{3\pi}{2} \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 0 & -3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\mathbf{\sin A = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}}$$

Q2] A) Evaluate $\int_c^0 \frac{dz}{z^3(z+4)}$, where c is the circle $|z| = 2$ **(6)**

SOLUTION:-

Poles are $z = 0$ and $z = -4$

$z = 0$ lies inside the circle

residue at $z = o$ $= \frac{1}{2!}\lim_{z\to 0}\left\{\frac{d^2}{dz^2}\left[z^3.\frac{1}{z^3(z+4)}\right]\right\} = \frac{1}{2}\lim_{z\to 0}\left\{\frac{d^2}{dz^2}\left[\frac{1}{(z+4)}\right]\right\}$

$= \frac{1}{2}\lim_{z\to 0}\left\{\frac{d}{dz}\left[-\frac{1}{(z+4)^2}\right]\right\} = \frac{1}{2}\lim_{z\to 0}\left\{\left[\frac{2(z+4)}{(z+4)^2}\right]\right\} = \frac{1}{64}$

Residue at $z = -4 = 0$ as it lies outside the circle.

$$\boldsymbol{\varphi f(z)dz = 2\pi i\left(\frac{1}{64}\right) = \frac{\pi i}{32}}$$

Q2] B) Memory capacity of 9 students was tested before and after a course of mediation for a month. State whether the course was effective or not from the data below. (6)

Before	**10**	**15**	**9**	**3**	**7**	**12**	**16**	**17**	**4**
After	**12**	**17**	**8**	**5**	**6**	**11**	**18**	**20**	**3**

SOLUTION :-

Before	10	15	9	3	7	12	16	17	4
After	12	17	8	5	6	11	18	20	3

Calculating difference between after – before,

X	2	2	-1	2	-1	-1	2	3	-1
$di = (X_i - 2)$	0	0	-3	0	-3	-3	0	1	-3
$di^2 = (X_i - 2)^2$	0	0	9	0	9	9	0	1	9

$$\bar{X} = a + \frac{\sum di}{n} = 2 + \frac{-11}{9} = 0.7778$$

$$\sum(X_i - \bar{X})^2 = \sum di^2 - \frac{(\sum di)^2}{n} = 37 - \frac{(-11)^2}{9} = 23.5556$$

$$S^2 = \frac{\sum(X_i - \bar{X})^2}{n} = \frac{23.5556}{9} = 2.6173$$

Null hypothesis ; $\mu = 0$

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n-1}} = \frac{0.7778 - 0}{\sqrt{2.6173}\,.\sqrt{8}}$$

t at $\alpha = 0.05$ for $v = 9 - 1 = 8$ is 2.306

$t_{cal} < t_{critical}$

Hypothesis is accepted

The students are not benefitted.

Q2] C) Solve the following LPP using simple method. (8)

Maximise ; $z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$

SOLUTION:-

Maximise ; $z = 6x_1 - 2x_2 + 3x_3$

Subject to $2x_1 - x_2 + 2x_3 \leq 2$

$x_1 + 4x_3 \leq 4$

$x_1, x_2, x_3 \geq 0$

Expressing in standard form;

$$z - 6x_1 + 2x_2 - 3x_3 + 0s_1 + 0s_2 = 0$$

$$2x_1 - x_2 + 2x_3 + s_1 + 0s_2 = 2$$

$$x_1 + 0x_2 + 4x_3 + 0s_1 + s_2 = 4$$

Iteration no	Basic variable	Coefficients of					RHS soln	ration
		x_1	x_2	x_3	s_1	s_2		
0	Z	-6	2	-3	0	0	0	
s_1 leaves	s_1	2	-1	2	1	0	2	1
x_1 enters	s_2	1	0	4	0	1	4	4
1	z	0	-1	3	3	0	6	
s_2 leaves	x_1	1	-1/2	1	1/2	0	1	-2
x_2 enters	s_2	0	1/2	3	-1/2	1	3	6
2	Z	0	0	9	2	2	12	
	x_1	1	0	4	0	1	4	
	x_2	0	1	6	-1	2	6	

$\mathbf{x_1 = 4;\ x_2 = 6;\ x_3 = 0\ \text{and}\ z_{max} = 12}$

Q3] A) Find the Eigen values and Eigen vectors of the following matrix, (6)

$$\mathbf{A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}}$$

SOLUTION:-

$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$$

Characteristics equation is given as follows:

$$\lambda^3 - 4\lambda^2 + (-1 - 6 + 6)\lambda + 4 = 0$$

$$\lambda^3 - 4\lambda^2 - \lambda + 4 = 0$$

$\lambda^2(\lambda - 4) - 1(\lambda - 4) = 0$

$(\lambda - 4)(\lambda^2 - 1) = 0$

$\lambda = 4$ and $\lambda = \pm 1$

For $\lambda = 4$

$$\begin{bmatrix} 0 & 6 & 6 \\ 1 & -1 & -2 \\ -1 & -4 & -7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0x_1 + 6x_2 + 6x_3 = 0$

$1x_1 - 1x_2 + 2x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -1 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{bmatrix} 0 & 6 \\ 1 & 2 \end{bmatrix}} = \frac{x^3}{\begin{bmatrix} 0 & 6 \\ 1 & -1 \end{bmatrix}} = t$$

$$\frac{x_1}{18} = \frac{x_2}{6} = \frac{x_3}{6} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3t \\ t \\ t \end{bmatrix}$$

For $\lambda = 1$

$$\begin{bmatrix} 3 & 6 & 6 \\ 1 & 2 & 2 \\ -1 & -4 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 + 2x_2 + x_3 = 0$

$-x_1 - 4x_2 - 4x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 2 & 1 \\ -4 & -4 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 1 & 1 \\ -1 & -4 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 1 & 2 \\ -1 & -4 \end{vmatrix}} = t$$

$$\frac{x_1}{-4} = \frac{x_2}{3} = \frac{x_3}{-2} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4t \\ -3t \\ -2t \end{bmatrix}$$

For λ = -1

$$\begin{bmatrix} 5 & 6 & 6 \\ 1 & 4 & 2 \\ -1 & -4 & -2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$5x_1 + 6x_2 + 6x_3 = 0$

$-x_1 - 4x_2 - 2x_3 = 0$

$$\frac{x_1}{\begin{vmatrix} 6 & 6 \\ -4 & -2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 5 & 6 \\ -1 & -2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 5 & 6 \\ -1 & -4 \end{vmatrix}} = t$$

$$\frac{x_1}{12} = \frac{x_2}{4} = \frac{x_3}{-14} = t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6t \\ 2t \\ -7t \end{bmatrix}$$

Eigen values = 4,1,-1

Eigen vectors = [3t t t],[-4t 3t -2t], [6t 2t -7t]

Q3] B) For a normal distribution 30% items are below 45% and 8% are above 64. Find the mean and variance of the normal distribution. (6)

SOLUTION:-

30% items below 45%

50-30 = 20% items are between 45 and m

8%are above 64

50-8= 42 % items are between m and 64

From table we find that

Area 0.2 corresponds to z = 0.525

Area 0.42 corresponds to z = 1.40

0.2 area is to the left of m

Hence z = -0.525

$$\frac{45-m}{\sigma} = -0.525; \frac{64-m}{\sigma} = 1.4$$

$45 - m = -0.525\sigma$

And $64 - m = 1.4\sigma$

We get, $\sigma = 9.87$

M = 50.1818

Mean = 50.1818; var =$\sigma^2 = 97.4169$

Q3] C) Obtain Laurent's series for $f(z) = \frac{1}{z(z+2)(z+1)}$ about $z = -2$ **(8)**

SOLUTION:-

$f(z) = \frac{1}{z(z+2)(z+1)}$ about z = -2

Applying partial fraction,

$$\frac{1}{z(z+2)(z+1)} = \frac{A}{z} + \frac{B}{z+2} + \frac{C}{z+1}$$

$$1 = A(z+2)(z+1) + B(z)(z+1) + C(z)(z+2)$$

Put z = -2

$$1 = B(-2)(-1) = B = \frac{1}{2}$$

Put z = 0

$$1 = 2A - 1, \quad A = \frac{1}{2}$$

Put z=-1

$$1 = -c, \quad c = 1$$

$$\frac{1}{z(z+2)(z+1)} = \frac{1}{2z} + \frac{1}{2(z+2)} - \frac{1}{z+1} = \frac{1}{2[(z+2)-2]} + \frac{1}{2(z+2)} - \frac{1}{[(z+2)-1]}$$

Let z+2 = u, we get,

$$= \frac{1}{2[u-2]} + \frac{1}{2u} - \frac{1}{u-1} = -\frac{1}{4\left(1-\frac{u}{2}\right)} + \frac{1}{2u} + \frac{1}{1-u}$$

$$\frac{1}{z(z+2)(z+1)} = \frac{1}{2u} - \frac{1}{4}\left[1 + \frac{u}{2} + \left(\frac{u}{2}\right)^2 + \cdots..\right] + [1 + u + u^2 \ldots..]$$

Q4] A) An ambulance services claims that it takes on an average 8.9 min to reach the destination in emergency calls. To check this the Licensing Agency has then timed on 50 emergency calls, getting a mean 0f 9.3 min with a S.D. 1.6 min. is the claim acceptable at 5% LOS? **(6)**

SOLUTION:-

$\overline{X} = 9.3$

$\mu = 8.9$

SD = 1.6 min

n = 50

we have $z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{9.3-8.9}{50/\sqrt{1.6}} = \frac{0.4\sqrt{1.6}}{50} = 0.0101$

$\alpha = 0.05$

Z at 5% level of significance = 1.96

0.0101 < 1.96

Thus the hypothesis is acceptable.

Q4] B) Using the Residue theorem Evaluate $\boldsymbol{\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta}$ **(6)**

SOLUTION:-

$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$

Put z = $e^{i\theta}$, $dz = ie^{i\theta} d\theta$

$dz = iz d\theta \quad d\theta = \frac{dz}{iz}$

And $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2} = \frac{z+\left(\frac{1}{z}\right)}{2}$

$\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta = \int_c^1 \frac{z^2}{5+4\left(\frac{z+\frac{1}{z}}{2}\right)} \frac{dz}{iz}$

$\int_0^{2\pi} \frac{e^{2i\theta}}{5+4\cos\theta} d\theta = \int_c^1 \frac{z^2}{i(2z^2+5z+2)} dz$

$c : |z| = 1$

Now the poles are given by $2z^2 + 5z + 2 = 0$

$(2z+1)(z+2) = 0$

$z = -\frac{1}{2} \quad \text{and} \quad -2$

for $z = -\frac{1}{2}$

Residue = $\lim_{z\to -1/2}\left(z+\frac{1}{2}\right)\frac{z^2}{2\left[z+\frac{1}{2}\right][z+2]i} = \frac{(-\frac{1}{2})^2}{2\left[-\left(\frac{1}{2}\right)+2\right]i} = \frac{1}{12i}$

Z = -2 lies outside the circle;

Residue = 0

$\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}\,d\theta = 2\pi i\left(\frac{1}{12i}\right) = \frac{\pi}{6}$

$\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}\,d\theta = \text{real part of } \int_0^{2\pi}\frac{e^{2i\theta}}{5+4\cos\theta}\,d\theta = \frac{\pi}{6}$

Q4] C) (i) If 10% of the rivets produced by a machine are defective, find the probability that out of 5 randomly chosen rivets at the most two will be defective.

(ii) If x denotes the outcome when a fair die is tossed, find M.G.F. of x and hence, find the mean and variance of x. (8)

SOLUTION:-

(1)

$p = 0.1$

$q = 1 - p = 0.9$

$n = 5$

P at most will be defective:

P(0 defective)+ P(1 defective) + P(2 defective)

$5C_0(0.1)^2(0.9)^5 + 5C_1(0.1)^1(0.9)^4 + 5C_2(0.1)^2(0.9)^3 = 0.59049 + 0.328 + 0.0792 = 0.99$

(2) Moment generating function,

$M_0(t) = E\left(e^{txi}\right) = \sum P_i e^{txi} = \frac{1}{6}e^t + \frac{1}{6}e^{2t} + \cdots \ldots\ldots\ldots.. \frac{1}{6}e^{6t}$

$= \frac{1}{6}(e^t + e^{2t} + \cdots \ldots\ldots.. e^{6t})$

$\mu_1' = \text{mean} = \frac{d}{dt[M_{0(t)}]} = \frac{1}{6}[e^t + 2e^{2t} + \cdots \ldots\ldots.. 6e^{6t}]$

$\text{at } t = 0; \text{mean} = \frac{1}{6}[1 + 2 + 3 \ldots \ldots . . +6] = \frac{21}{6} = \frac{7}{2}$

$\text{variance} = \mu_2' = \frac{d^2}{dt^2}\left[M_{0(t)}\right] = \frac{1}{6}[e^t + 4e^{2t} + \cdots \ldots \ldots . . 36e^{6t}]$

$= \frac{1}{6}[1 + 4 + 9 \ldots \ldots . +36] = \frac{91}{6}$

$\mu_2 = \mu_2' - {\mu_1'}^2 = \frac{91}{6} - \frac{49}{4} = \frac{35}{12}$

$\mathbf{mean} = \frac{7}{2}$ **and variance** $= \frac{35}{12}$

Q5] A) Check whether the following matrix is Derogatory or Non- Derogatory

$\mathbf{A} = \begin{bmatrix} \mathbf{6} & \mathbf{-2} & \mathbf{2} \\ \mathbf{-2} & \mathbf{3} & \mathbf{-1} \\ \mathbf{2} & \mathbf{-1} & \mathbf{3} \end{bmatrix}$ **(6)**

SOLUTION:-

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Characteristics equation :

$\lambda^3 - 12\lambda^2 + (8 + 14 + 14)\lambda - 32 = 0$

Λ = 8,2,2

Let us assume $(x - 8)(x - 2) = x^2 - 10x + 16$ annihilates A

Now $A^2 - 10A + 16I$

$= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - 10\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + 16\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -16 & 0 & 0 \\ 0 & -16 & 0 \\ 0 & 0 & -16 \end{bmatrix} + \begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\mathbf{x^2 - 10x + 16}$ **annihilates A thus f(x) is the monic polynomial of lowest degree**

$\mathbf{minimal\ polynomial} = \mathbf{x^2 - 10x + 16}$

Q5] B)Justify if there is any relationship between sex and color for the following data.

(6)

colour	male	female	Total
Red	**10**	**40**	**50**
White	**70**	**30**	**100**
Green	**30**	**20**	**50**
total	**110**	**90**	**200**

SOLUTION :-

colour	male	female	Total
Red	10	40	50
White	70	30	100
Green	30	20	50
total	110	90	200

Null hypothesis, H_0, there is no relationship

Alternative hypothesis , H_a there is an relationship

Calculation of test statistics;

Red and male : $\frac{110\times50}{200} = 27.5$

White and male : $\frac{110\times100}{200} = 55$

Green and male : $\frac{110\times50}{200} = 27.5$

Red and female : $\frac{90\times50}{200} = 22.5$

White and female : $\frac{90\times100}{200} = 45$

Green and female : $\frac{90\times50}{200} = 22.5$

O	E	$(O-E)^2$	$(O-E)^2/E$
27.5	10	306.25	30.625
55	70	225	3.2143

27.5	30	6.25	0.2083
22.5	40	306.25	7.6562
45	30	225	7.5
22.5	20	6.25	0.3125 X = 49.5288

$\alpha = 0.05$

Degree of freedom = $(r-1)(c-1) = (3-1)(2-1) = 2$

Critical value = 5.991

$X^2_{calc} \;>\; X^2_{table}$

thus null hypothesis is rejected

There is relationship between colour gender.

Q5] C) Use the dual simplex method to solve the following L.P.P. (8)

Minimise

$\mathbf{z = 2x_1 + x_2}$

Subject to $\mathbf{3x_1 + x_2 \geq 3}$

$\mathbf{4x_1 + 3x_2 \geq 6}$

$\mathbf{x_1 + 2x_2 \leq 3}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION :-

Minimise

$z = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3$

$x_1, x_2 \geq 0$

$-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq -\ 6$

$x_1 + 2x_2 \leq 3$

Introducing stack variables

$Z' - 2x_1 - x_2 - 0s_1 - 0s_2 - 0s_3$

$-3x_1 - x_2 + s_1 - 0s_2 - 0s_3 = \ -6$

$x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 = 3$

Iterations no	Basics variables	Coefficients of					RHS Solution
		x_1	x_2	s_1	s_2	s_3	
0	Z'	-2	-1	0	0	0	0
s_2 leaves	s_1	-3	-1	1	0	0	-3
x_2 enters	s_2	-4	-3	0	1	0	-6
	s_3	1	2	0	0	1	3
Ratio:		-1/2	-1/3				
1	Z'	2	2	0	-1	0	6
	s_1	-5/3	0	0	1/3	0	-1
	x_2	4/3	1	0	1/3	0	2
	s_3	-5/3	0	0	-2/3	1	-1
Ratio:		-6/5			3/2	0	
2		11/3	2	0	-1/3	0	7
s_3 leaves	s_1	0	0	0	1	-1	0
x_1 enters	x_2	0	1	0	3	-4/3	6/5
	x_1	1	0	0	2/5	-3/5	3/5

$x_1 = \frac{3}{5}; \ \ x_2 = \frac{6}{5};$

$Z_{min} = 2\left(\frac{3}{5}\right) + \frac{6}{5} = \frac{12}{5}$

$\mathbf{Z_{min}} = \frac{\mathbf{12}}{\mathbf{5}}$

Q6] A)Show that the matrix A satisfies Cayley-Hamilton theorem and hence find A^{-1} where **(6)**

$$\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-2} \\ \mathbf{-1} & \mathbf{3} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} & \mathbf{1} \end{bmatrix}$$

SOLUTION :-

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

Characteristics equation,

$$\lambda^3 - 5\lambda^2 + (3 + 1 + 5)\lambda - 1 = 0$$

$$\lambda^3 - 5\lambda^2 + (9)\lambda - 1 = 0$$

Cayley Hamilton theorem states that;

$$A^3 - 5A^2 + 9A - I = 0 \qquad \text{..................... (1)}$$

$$A^2 = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}^2 = \begin{bmatrix} -1 & 12 & -4 \\ -4 & 7 & 2 \\ 2 & -8 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}^3 = \begin{bmatrix} -13 & 42 & -2 \\ -11 & 9 & 10 \\ 10 & -22 & -3 \end{bmatrix}$$

Substituting the values in equation (1)

We get, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Multiply the above equation by A^{-1}

$$A^{-1} = A^2 - 5A + 9I$$

$$A^{-1} = \begin{bmatrix} -1 & 12 & -4 \\ -4 & 7 & 2 \\ 2 & -8 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$\mathbf{A^{-1}} = \begin{bmatrix} \mathbf{3} & \mathbf{2} & \mathbf{6} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{2} & \mathbf{5} \end{bmatrix}$$

Q6] B) The probability Distribution of a random variable X is given by (6)

X	**-2**	**-1**	**0**	**1**	**2**	**3**
P(X=x)	**0.1**	**k**	**0.2**	**2k**	**0.3**	**k**

Find k, mean and variance.

SOLUTION:-

X	-2	-1	0	1	2	3
P(X=x)	0.1	k	0.2	2k	0.3	k

$\sum P_i = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$0.6 + 4k = 1$

$4k = 0.4$

$k = 0.1$

X	-2	-1	0	1	2	3
P(X=x)	0.1	0.1	0.2	0.2	0.3	0.1

$\text{Mean} = E(X) = \sum P_i x_i = -0.2 - 0.1 + 0.2 + 0.6 + 0.3 = 0.8$

$E(X^2) = \sum P_i {x_i}^2 = 0.4 + 0.1 + 0.2 + 1.2 + 0.9 = 2.8$

$\text{Variance} = E(X^2) - [E(X)]^2 = 2.8 - 0.64 = 2.16$

$\mathbf{E(X^2) = 2.8}$

Variance = 2.16

Q6] C)Using Kuhn- Tucker conditions solve the following NLPP (8)

Maximise : $\mathbf{Z = 2x_1 - 7x_2 + 12x_1x_2}$

Subject to : $\mathbf{2x_1 + 5x_2 \leq 98}$

$\mathbf{x_1, x_2 \geq 0}$

SOLUTION:-

Maximise : $Z = 2x_1 - 7x_2 + 12x_1x_2$

Subject to : $2x_1 + 5x_2 \leq 98$

$x_1, x_2 \geq 0$

We rewrite the given problem as:

$f(x) = 2x_1^2 - 7x_2^2 + 12x_1x_2$

$h(x_1x_2) = 2x_1 + 5x_2 - 98$

Kuhn tucker conditions are:

$\frac{\partial f}{\partial x_1} - \frac{\lambda \partial h}{\partial x_1} = 0; \frac{\partial f}{\partial x_2} - \frac{\lambda \partial h}{\partial x_2} = 0$

$\lambda h(x_1, x_2) = 0;\ \ h(x_1, x_2) \leq 0,\ \lambda \geq 0$

We get,

$4x_1 + 12x_2 - \lambda(2) = 0$ ………………… (1)

$12x_1 - 14x_2 - \lambda(5) = 0$ ……………………(2)

$\lambda(2x_1 + 5x_2 - 98) = 0$ ………………… (3)

$2x_1 + 5x_2 - 98 \leq 0$ ……………… (4)

$\lambda \geq 0$ ………… (5)

From (3) we get either $\lambda = 0$ or $(2x_1 + 5x_2 - 98) = 0$

Case 1: $\lambda = 0$ and $(2x_1 + 5x_2 - 98) \neq 0$

From 1 and 2,

$4x_1 + 12x_2 = 0$

$12x_1 - 14x_2 = 0$

On solving simultaneously we get $x_1 = x_2 = 0$

Case 2:

$\lambda \neq 0$ and $2x_1 + 5x_2 - 98 = 0$

$4x_1 + 12x_2 - \lambda(2) = 0$

$12x_1 - 14x_2 - \lambda(5) = 0$

$\lambda = \frac{12x_1 - 14x_2}{5}$

Equation I : $4x_1 + 12x_2 - 2\left[\frac{12x_1 - 14x_2}{5}\right] = 0$

$20x_1 + 60x_2 - 24x_1 + 28x_2 = 0$

$-4x_1 + 88x_2 = 0$ (divide through by 4)

$-x_1 + 22x_2 = 0$

Put $x_1 = 22x_2$ in 4

$2(22x_2) + 5x_2 = 98$

$44x_2 + 5x_2 = 98$

$49x_2 = 98$

$x_2 = 2$

$2x_1 + 10 = 98$

$2x_1 = 88$

$x_1 = 44$

These values satisfy all conditions,

$Z_{max} = 2(1936) - 7(4) + 12(44)(2) = 4900$

$\mathbf{Z_{max} = 4900}$