COMPUTER ENGINEERING

APPLIED MATHEMATICS – 3

SEM – 3(CBCGS MAY – 2019)

Q1] a) Find the Laplace transform of te^t sin2t cost **(5)**

Solution:-

$$L[\sin 2t \,.\, \cos t] = L\left[\frac{1}{2}\{\sin 3t+\sin t\}\right] = \frac{1}{2}L[\sin 3t+\sin t]$$

$$L[\sin 2t \,.\, \cos t] = \frac{1}{2}\left[\frac{3}{s^2+9}+\frac{1}{s^2+1}\right]$$

$$L[\sin 2t \,.\, \cos t] = (-1)^n\frac{1}{2}\frac{d}{ds}\left[\frac{3}{s^2+9}+\frac{1}{s^2+1}\right] = \frac{-1}{2}\left[3\left\{\frac{-2s}{(s^2+9)^2}\right\}-\left\{\frac{2s}{(s^2+1)^2}\right\}\right]$$

$$L[\sin 2t \,.\, \cos t] = \frac{1}{2}\left[\frac{6s}{(s^2+9)^2}+\frac{2s}{(s^2+1)^2}\right] = \frac{3s}{(s^2+9)^2}+\frac{s}{(s^2+1)^2} = \varphi(s)$$

$$L\left[te^t\sin 2t \,.\, \cos t\right] = \varphi(s-a)$$

By first shifting theorem;

$$L\left[te^t\sin 2t \,.\, \cos t\right] = \frac{3(s-1)}{[(s-1)^2+9]^2}+\frac{(s-1)}{[(s-1)^2+1]^2} = \frac{3(s-1)}{[s^2-2s+10]^2}+\frac{(s-1)}{[s^2-2s+2]^2}$$

$$L\left[te^t\sin 2t \,.\, \cos t\right] = \frac{3(s-1)}{[s^2-2s+10]^2}+\frac{(s-1)}{[s^2-2s+2]^2}$$

Q1] b) Find the inverse Laplace transform of $\frac{s+2}{s^2(s+3)}$ **(5)**

Solution:-

$$\frac{s+2}{s^2(s+3)} = \frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+3}$$

$$s + 2 = As^2(s+3) + B(s+3) + Cs^2$$

Put s = 0;

$2 = B(3) \Rightarrow \quad B = \frac{2}{3}$

Put s = -3;

$-1 = C(-3)^2 \quad \Rightarrow \quad C = -\frac{1}{9}$

Comparing coefficient of s^2 on both sides;

$0 = A + C$

$A = \frac{1}{9}$

$$L^{-1}\left[\frac{s+2}{s^2(s+3)}\right] = \frac{1}{9}L^{-1}\left[\frac{1}{s}\right] + \frac{2}{3}L^{-1}\left[\frac{1}{s^2}\right] - \frac{1}{9}L^{-1}\left[\frac{1}{s+3}\right] = \frac{1}{9}(1) + \frac{2}{3}t - \frac{1}{9}e^{-3t}$$

$$L^{-1}\left[\frac{s+2}{s^2(s+3)}\right] = \frac{1}{9} + \frac{2}{3}t - \frac{1}{9}e^{-3t}$$

Q1] c) Determine whether the function $f(z) = x^2 - y^2 + 2ixy$ is analytic and if so find its derivative (5)

Solution:-

$f(z) = x^2 - y^2 + 2ixy$

$u + iv = (x^2 - y^2) + i(2xy)$

$u = (x^2 - y^2)$ and $v = 2xy$

Differentiating u & v partially wrt x & y

$u_x = 2x; \quad u_y = -2y$

$v_x = 2y; \quad v_y = 2x$

$u_x = v_y \quad \& \quad u_y = -v_x$

CR equation are satisfied

Hence f(z) is analytic.

$f'(z) = u_x + iv_x = 2x + i2y$

Derivative of $f(z) = 2(x+iy) = 2z$

Q1] d) Find the Fourier series for $f(x) = e^{-|x|}$ in the interval $(-\pi,\pi)$ (5)

Solution:-

Fourier series for $f(x) = e^{-|x|}$ in $(-\pi,\pi)$

$f(x) = e^{-|x|}$

$f(-x) = e^{-|-x|} = e^{-|x|} = f(x)$

Hence f(x) is an even function and hence $b_n = 0$

Also, $f(x) = \begin{cases} e^{x}, & -\pi<x<0 \\ e^{-x}, & 0<x<\pi \end{cases}$

$$a_0 = \frac{1}{\pi}\int_0^{\pi} f(x)dx = \frac{1}{\pi}\int_0^{\pi} e^{-x}dx = \frac{1}{\pi}[-e^{-x}]_0^{\pi} = \frac{1}{\pi}[-e^{-\pi}-(-1)] = \frac{1}{\pi}[1-e^{-\pi}]$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\cos nx\,dx = \frac{2}{\pi}\int_0^{\pi} e^{-x}\cos nx\,dx$$

$$a_n = \frac{2}{\pi}\left[\frac{e^{-x}}{1+n^2}(-\cos nx+n\sin nx)\right]_0^{\pi} = \frac{2}{\pi}\left[\frac{e^{-x}}{1+n^2}\{-\cos nx+n\sin nx\}-\frac{1}{1+n^2}(-\cos 0+n\sin 0)\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[\frac{-e^{-x}\cos n\pi+1}{(1+n^2)}\right] = \frac{2(-e^{-x}(-1)^n+1)}{\pi(1+n^2)}$$

$$f(x) = \frac{1}{\pi}(1-e^{-\pi}) + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{1-e^{-x}(-1)^n}{(1+n^2)}\right)\cos nx$$

Q2] a) Evaluate $\int_0^{\infty}\frac{e^{-t}-\cos t}{te^{4t}}\,dt$ (6)

Solution:-

$$L[e^{-t}-\cos t] = L\left[\frac{1}{s+1} - \frac{s}{s^2+1}\right]$$

$$L\left[\frac{e^{-t}-\cos t}{te^{4t}}\right] = \int_s^{\infty}\frac{1}{s+1}-\frac{s}{s^2+1}\,ds = [\ln(s+1)-\frac{1}{2}\ln(s^2+1)]_s^{\infty}$$

$$L\left[\frac{e^{-t}-\cos t}{te^{4t}}\right] = [\ln(s+1)-\frac{1}{2}\ln\sqrt{(s^2+1)}]_s^{\infty} = [\ln\left(\frac{(s+1)}{\sqrt{(s^2+1)}}\right)]_s^{\infty}$$

$$L\left[\frac{e^{-t}-\cos t}{te^{4t}}\right] = [\ln\left(\frac{\left(1+\frac{1}{s}\right)}{\sqrt{(1+\frac{1}{s^2})}}\right)]_s^{\infty} = \ln(1)-\ln\left(\frac{(s+1)}{\sqrt{(s^2+1)}}\right) = -\ln\left(\frac{(s+1)}{\sqrt{(s^2+1)}}\right)$$

$$L\left[\frac{e^{-t}-\cos t}{te^{4t}}\right] = \ln\left(\frac{(s+1)}{\sqrt{(s^2+1)}}\right)$$

$$\int_0^{\infty}e^{-st}\left[\frac{e^{-t}-\cos t}{t}\right]dt=\ln\left(\frac{\sqrt{(s^2+1)}}{(s+1)}\right)$$

Put s = 4

$$\int_0^{\infty}\frac{e^{-t}-\cos t}{te^{4t}} =\ln\left(\frac{\sqrt{(16+1)}}{(4+1)}\right) = \ln\left(\frac{\sqrt{(17)}}{(5)}\right)$$

Q2] b) Find the Z – transform of $f(k) = f(x) = \begin{cases}3^k, & k<0\\ 2^k, & k\geq 0\end{cases}$ **(6)**

Solution:-

$$Z\{f(k)\} = \sum_{k=-\infty}^{-1} 3^k z^{-k} + \sum_{k=-0}^{\infty} 2^k z^{-k}$$

$$Z\{f(k)\} = \sum_{n=1}^{\infty} 3^{-n}z^{n} + \left\{1+\frac{2}{z}+(\frac{2}{z})^2+.........\right\}$$ [put –k = n in 1st series]

$$Z\{f(k)\} = \left\{\frac{z}{3}+\left(\frac{z}{3}\right)^2+\left(\frac{z}{3}\right)^3+...\right\} + \left\{1+\frac{2}{z}+(\frac{2}{z})^2+.........\right\}$$

$$Z\{f(k)\} = \frac{z}{3}\left(1+\frac{z}{3}+\left(\frac{z}{3}\right)^2+.....\right) + \frac{1}{1-\frac{2}{z}} = \frac{z}{3}\left(\frac{1}{1-\frac{z}{3}}\right) + \frac{1}{z-2}z$$

$$Z\{f(k)\} = \frac{z}{3-z} + \frac{z}{z-2} = \frac{z(z-2)+z(3-z)}{(3-z)(z-2)} = \frac{z^2-2z+3z-z^2}{(3-z)(z-2)}$$

$$Z\{f(k)\} = \frac{z}{(3-z)(z-2)}$$

Q2] c) Show that the function u = 2x(1 - y) is a harmonic function . Find its harmonic conjugate and corresponding analytic function (8)

Solution:-

$u = 2x - 2xy$

$u_x = 2 - 2y \qquad u_y = -2x$

$u_x^2 = 0 \qquad u_y^2 = 0$

$u_x^2 + u_y^2 = 0$ laplace equation is satisfied

U is harmonic

$u_x = \varphi_1(x,y) = 2 - 2y$

$u_y = \varphi_1(x,y) = -2x$

$\varphi_1(z,0) = 2 - 2(0) = 2$

$\varphi_2(z,0) = -2z$

By Milne Thompsons method;

$f(z) = \int u_x - iu_z dz = \int 2 - i(-2z)dz = 2\int(1+z)dz = 2\left[z + \frac{z^2}{2}\right]$

$f(z) = 2z + z^2$

$u + iv = 2(x+iy) + (x+iy)^2$

$u + iv = 2x + 2iy + x^2 - y^2 + 2ixy$

On comparing imaginary part,

$v = 2y + 2xy$

$v = 2y(1+x)$

Q3] a) Find the equation of the line of regression of y on x for the following data (6)

X	10	12	13	16	17	20	25
Y	19	22	24	27	29	33	37

Solution:-

X	Y	X^2	Y^2	XY
10	19	100	361	190
12	22	144	484	264
13	24	169	576	312
16	27	256	729	432
17	29	289	841	493
20	33	400	1089	660
25	37	625	1369	925
$\sum X = 113$	181	1983	5449	3276

$\sum X = 113$; $\sum Y = 181$; $\sum X^2 = 1983$; $\sum Y^2 = 5449$; $\sum XY = 3276$

Here N = 7

Line of regression y on x => y = a + bx

The normal equation are:

$\sum y = Na + b\sum x$; 181 = 7a + 113b

$\sum xy = \sum x + b\sum x^2$; 3276 = 113a + 1983b

On solving simultaneously;

a = - 10.1304 and b = 2.2293

line of regression ; y on x => y = - 10.1304 + 2.2293b

Q3] b) Find the bilinear transformation which maps z = 2, $1, 0$ onto w = 1, $0, i$ (6)

Solution:-

Let the transformation be $\omega = \frac{az+b}{cz+d}$ ……………(1)

Putting the given values of z & ω we get $1 = \frac{2a+b}{2c+a}$; $0 = \frac{a+b}{c+d}$; $i = \frac{b}{d}$

From third we get b = di

From second we get a = $-b = di$

$2c + 2d = 2a + b$

We get $2c + d = -2di + di$

$2c = -di - d$

$c = -\frac{(i+1)d}{2}$

Hence from 1; we get

$$\omega = \frac{-diz+di}{\left\{-\frac{(i+1)dz}{2}\right\}+d} = \frac{2(-iz+i)}{-(i+1)+2}$$

$$\omega = \frac{2(z-1)}{(1-i)z+2i}$$

Q3] c) Obtain the expansion of $f(x)$ = $x(\pi-x)$, $0 < x < \pi$ as a half range cosine series. Hence show that $\sum_1^\infty \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$ (8)

Solution:-

Cosine series

$f(x) = a_0 + \sum a_n \cos nx$

$$a_0 = \frac{1}{\pi}\int_0^\pi \pi x - x^2 dx = \frac{1}{\pi}\left[\frac{\pi x^2}{2} - \frac{x^3}{3}\right]_0^\pi = \frac{1}{\pi}\left[\frac{\pi^2}{2} - \frac{\pi^3}{3}\right] = \frac{\pi^2}{6} \quad \text{...... (1)}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}(\pi x - x^2)\cos nx\,dx$$

$$a_n = \frac{2}{\pi}\left[(\pi x - x^2)\frac{\sin(nx)}{n} - (\pi - 2x)\left(\frac{-\cos nx}{n^2}\right) + (-2)\left(\frac{-\sin(nx)}{n^3}\right)\right]_0^{\pi}$$

$$a_n = \frac{2}{\pi}\left[\left\{0 - \frac{\pi\cos\pi}{n^2} + 0\right\} - \left\{0 + \frac{\pi\cos n(0)}{n^2}\right\} + 0\right] = \frac{2}{\pi}\left[\frac{-\pi(-1)^n}{n^2} - \frac{\pi}{n^2}\right] = \frac{-2}{n^2}\left[(-1)^n + 1\right]$$

$$f(x) = \frac{\pi^2}{6} - 2\sum_{n=1}^{\infty}\frac{(-1)^n + 1}{n^2}\cos nx$$

$$x(\pi - x) = \frac{\pi^2}{6} - 2\sum_{n=1}^{\infty}\frac{(-1)^n + 1}{n^2}\cos nx$$

Put x = 0

$$0 = \frac{\pi^2}{6} - 2\sum_{n=1}^{\infty}\frac{(-1)^n + 1}{n^2}\cos nx$$

$$-\frac{\pi^2}{6} = -2\sum_{n=1}^{\infty}\frac{(-1)^n + 1}{n^2}$$

$$\frac{\pi^2}{12} = \sum_{n=1}^{\infty}\frac{(-1)^n + 1}{n^2}$$

Q4] a) Find the inverse Laplace Transform by using convolution theorem

$$\frac{1}{(s^2+1)(s^2+9)}$$ **(6)**

Solution:-

Let $\varphi_1(s) = \frac{1}{(s^2+1)}$ and $\varphi_2(s) = \frac{1}{(s^2+9)}$

$$L^{-1}[\varphi_1(s)] = L^{-1}\left[\frac{1}{(s^2+1)}\right] = \sin t$$

$$L^{-1}[\varphi_2(s)] = L^{-1}\left[\frac{1}{(s^2+9)}\right] = \frac{1}{3}\sin 3t$$

$$L^{-1}\left[\frac{1}{(s^2+1)}\cdot\frac{1}{(s^2+9)}\right] = \int_0^{l}\sin u.\frac{1}{3}\sin 3(l-u)\,du$$

$$L^{-1}\left[\frac{1}{(s^2+1)}\cdot\frac{1}{(s^2+9)}\right] = -\frac{1}{6}\int_0^t \cos[(1-3)u+3t]-\cos[(4u-3t)]du$$

$$L^{-1}\left[\frac{1}{(s^2+1)}\cdot\frac{1}{(s^2+9)}\right] = -\frac{1}{6}\int_0^t \cos(3t-2u)-\cos(4u-3t)du$$

$$L^{-1}\left[\frac{1}{(s^2+1)}\cdot\frac{1}{(s^2+9)}\right] = -\frac{1}{6}\left[\frac{\sin(3t-2u)}{-2}-\frac{\sin(4u-3t)}{4}\right]_0^t = -\frac{1}{6}\left[\frac{-\sin t}{2}-\frac{\sin t}{4}-\left(\frac{-\sin 3t}{2}-\frac{\sin(-3t)}{4}\right)\right]$$

$$L^{-1}\left[\frac{1}{(s^2+1)}\cdot\frac{1}{(s^2+9)}\right] = -\frac{1}{6}\left[\frac{\sin 3t}{2}-\frac{\sin 3t}{4}-\frac{\sin t}{2}-\frac{\sin t}{4}\right]$$

$$L^{-1}\left[\frac{1}{(s^2+1)}\cdot\frac{1}{(s^2+9)}\right] = -\frac{1}{6}\left[\frac{2\sin 3t-\sin 3t}{4}-\frac{2\sin t-\sin t}{4}\right]$$

$$L^{-1}\left[\frac{1}{(s^2+1)}\cdot\frac{1}{(s^2+9)}\right] = \frac{1}{24}[3\sin t-\sin 3t]$$

Q4] b) Calculate the coefficient of correlation between Price and Demand (6)

Price : 2, 3, 4, 7, 4

Demand : 8, 7, 3, 1, 1.

Solution:-

x	y	x^2	y^2	xy
2	8	4	64	16
3	7	9	49	21
4	3	16	9	12
7	1	49	1	7
4	1	16	1	4
∑N =5 ; 20	20	94	124	60

$$\bar{X} = \frac{\sum x}{N} = \frac{20}{5} = 4$$

$$\bar{Y} = \frac{\sum y}{N} = \frac{20}{5} = 4$$

$$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}} = \frac{60 - \frac{400}{5}}{\sqrt{\left(94 - \frac{400}{5}\right)\left(124 - \frac{400}{5}\right)}} = -0.8058$$

Q4] c) Find the inverse Z- transform for the following: (8)

1. $\frac{z}{z-5}$, $|z| < 5$ **2.** $\frac{1}{(z-1)^2}$, $|z| > 1$

Solution:-

1. $\frac{z}{z-5}$, $|z| < 5$

$$\frac{-z}{5(1-\frac{z}{5})} = -\frac{z}{5}\left[1-\frac{z}{5}\right]^{-1} = -\frac{z}{5}\left[1+\left(\frac{z}{5}\right)+\left(\frac{z}{5}\right)^2+.........\right]$$

$$= -\left[\left(\frac{z}{5}\right)+\left(\frac{z}{5}\right)^2+\left(\frac{z}{5}\right)^3.........\right]$$

Coefficient of $z^n = -5^n$

Put $n = -k$; $n \geq 1$

$z^{-k} = -5^{-k}$; $k \leq -1$

$Z^{-1}[F(z)] = -5^{-k}$; $k \leq -1$

2. $\frac{1}{(z-1)^2}$, $|z| > 1$

$$\frac{1}{z^2(1-\frac{1}{z})^2} = \frac{1}{z^2}\left[1-\frac{1}{z}\right]^{-2} = \frac{1}{z^2}\left[1+2\left(\frac{1}{z}\right)+3\left(\frac{1}{z}\right)^2+ .....\right]$$

$$= \left[\frac{1}{z^2}+\frac{2}{z^3}+\frac{3}{z^4}+... \frac{(n-1)}{z^n}\right]$$

Coefficient of $z^{-n} = n - 1$; $n \geq 2$

Put n = k

$z^{-k} = k - 1 \;\; ; \; k \geq 2$

$z^{-1}[f(z)] = k - 1 \; ; k \geq 2$

Q5] a) Find the Laplace transform of e^{-t}sint $H(t-\pi)$ (6)

Solution:-

Here f(t) = e^{-t}sint and a = π

$f(t+\pi) = e^{-(t+\pi)}\sin(t+\pi) \;=\; -e^{-(t+\pi)}\sin t \;=\; -e^{-(\pi)}.e^{-(t)}\sin t$

$$L[f(t+\pi)] = L\left[-e^{-\pi}.e^{-(t)}.\sin t\right] = -e^{-\pi}L\left[e^{-(t)}\sin t\right] = -e^{-\pi}\left[\frac{1}{(s+1)^2+1}\right]$$

$$L[f(t+\pi)H(t-\pi)] = -e^{-\pi s}.e^{-\pi}.\frac{1}{s^2+2s+2} = -e^{-\pi(s+1)}.\frac{1}{(s+1)^2+1}$$

$$L\left[e^{-t}\sin t\, H(t-\pi)\right] = \frac{-e^{-\pi(s+1)}}{s^2+2s+2}$$

Q5] b) Show that the set of functions {sinx, sin3x, sin5x, ……} is orthogonal over [0,π/2] . Hence construct orthogonal set of functions. (6)

Solution:-

We have $f_n(x) = \sin(nx)$

$$\int_0^{\pi/2} f_m(x).f_n(x)dx = \int_0^{\pi/2}\sin mx.\sin nx\, dx$$

$$\int_0^{\pi/2} f_m(x).f_n(x)dx = -\frac{1}{2}\int_0^{\pi/2}\sin(m+n)x-\sin(m-n)x\, dx$$

$$\int_0^{\pi/2} f_m(x).f_n(x)dx = -\frac{1}{2}\left[\frac{-\cos(m+n)x}{(m+n)} - \left\{\frac{-\cos(m-n)x}{(m-n)}\right\}\right]_0^{\pi/2}$$

$$\int_0^{\pi/2} f_m(x).f_n(x)dx = \frac{1}{2}\left[\frac{\cos(m+n)x}{(m+n)} - \frac{\cos(m-n)x}{(m-n)}\right]_0^{\pi/2}$$

Now two cases arises;

1. $m \neq n$

$$= \frac{1}{2}\left[\frac{\cos(m+n)\pi/2}{(m+n)} - \frac{\frac{\cos(m-n)\pi}{2}}{(m-n)} - \{1-1\}\right] = 0$$

for $m = n$

$$\int_0^{\pi/2} \sin^2(2n+1)dx = \int_0^{\pi/2} \frac{1-\cos 2(2n+1)x}{2} = \frac{1}{2}\left[x - \frac{\sin 2(2n+1)x}{2(2n+1)}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left[\frac{\pi}{2} - 0 - \{0-0\}\right]_0^{\pi/2} = \frac{\pi}{4} \neq 0$$

$$\int_0^{\pi/2} \begin{matrix} f_m(x)f_n(x)dx = 0 & \text{if } m \neq n \\ f_m(x)f_n(x)dx \neq 0 & \text{if } m = n \end{matrix}$$

given set of functions is orthogonal in $[0, \frac{\pi}{2}]$

$$\int_0^{\pi/2} [f_n(x)]^2 dx = \frac{\pi}{4}$$

Divide by $\frac{\pi}{4}$ on both sides;

$$\int_0^{\pi/2} \frac{4}{\pi}[f_n(x)]^2 dx = \frac{4}{\pi}.\frac{\pi}{4}$$

i.e. $\int_0^{\frac{\pi}{2}} \frac{2}{\sqrt{\pi}} f_n(x).\frac{2}{\sqrt{\pi}} f_n(x)dx = 1$

this is orthonormal set, $\varphi_n(x) = \frac{2}{\sqrt{\pi}}\sin(2n+1)x$

Q5] c) Solve using Laplace transform $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^t$, given that y(0) = 4 and y'(0) = 2 **(8)**

Solution:-

Let $L(y) = y$

Taking laplace transform on both sides

$$L(y'') + 2L(y') + L(y) = L(3te^{-t})$$

But , $L(y') = S\bar{y} - y(0) = S\bar{y} - 4$

$$L(y'') = S^2y - Sy(0) - y'(0) = S^2y - 4s - 2$$

$$L(e^{-t}) - \frac{1}{s+1}$$

$$L[te^{-t}] = \frac{-d}{ds}\left(\frac{1}{s+1}\right) = \frac{1}{(s+1)^2}$$

The equation becomes,

$$\left(s^2\bar{y} - 4s - 2\right) + 2\left(s\bar{y} - 4\right) + \bar{y} = 3.\frac{1}{(s+1)^2}$$

$$\left(s^2 - 4s - 2\right)\bar{y} - 4s - 10 = \frac{3}{(s+1)^2}$$

$$(s+1)^2\bar{y} = 4s + 10 + \frac{3}{(s+1)^2}$$

$$\bar{y} = \frac{3}{(s+1)^4} + \frac{4s+10}{(s+1)^2}$$

$$\bar{y} = \frac{3}{(s+1)^4} + \frac{4s+10}{(s+1)^2} + \frac{6}{(s+1)^2}$$

$$\bar{y} = \frac{3}{(s+1)^4} + \frac{4}{(s+1)} + \frac{6}{(s+1)^2}$$

Take inverse on both sides

$$y = L^{-1}\left[\frac{3}{(s+1)^4}\right] + 4L^{-1}\left[\frac{1}{s+1}\right] + 6L^{-1}\left[\frac{1}{(s+1)^2}\right]$$

$$y = 3e^{-t}.\frac{t^3}{3!} + 4e^{-t} + 6e^{-t}t$$

Q6] a) Find the complex form of Fourier series for $f(x) = 3x$ in $(0, 2\pi)$ (6)

Solution:-

$$f(x) = \sum_{-\infty}^{\infty} C_n e^{inx}$$

$$C_n = \frac{1}{2\pi}\int_0^{2\pi} f(x)e^{inx}dx = \frac{1}{2\pi}\int_0^{2\pi} 3xe^{inx}dx = \frac{3}{2\pi}\int_0^{2\pi} xe^{inx}dx$$

$$C_n = \frac{3}{2\pi}\left[\frac{xe^{-inx}}{-in} - \frac{e^{-inx}}{(-in)^2}\right]_0^{2\pi} = \frac{3}{2\pi}\left[\frac{-2\pi e^{-inx}}{in} + \frac{e^{-in2\pi}}{n^2} - \left\{0 + \frac{1}{n^2}\right\}\right] = -\frac{3}{in}$$

$$C_n = -\frac{3}{in}\cdot\frac{i}{i} = \frac{3i}{n};\quad n \neq 0$$

$$C_0 = 3\pi$$

$$f(x) = C_0 + \sum_{-\infty}^{\infty} C_n e^{inx}$$

$$f(x) = 3\pi + 3i\sum_{-\infty}^{\infty} e^{inx}/n$$

Q6] b) If f(z) is an analytic function with constant modulus then, prove that f(z) is constant (6)

Solution:-

Let $f(z) = u + iv$

But $|f(z)| = C$

$| u + iv| = C$

$u^2 + v^2 = C^2$

Differentiate it partially wrt x and y

$uu_x + vv_x = 0$

$uu_y + vv_y = 0$

f(z) is analytic , $u_x = u_y$ and $u_y = -v_x$

$uu_x - vv_y = 0$ and $uu_y + vv_x = 0$

$uu_x = vu_y \quad \Rightarrow \quad u_y = \frac{uu_x}{v}$

$u\left(\frac{uu_x}{v}\right) + vu_x = 0 \quad \Rightarrow \quad (u^2+v^2)u_x = 0$

Eliminating u_y; $(u^2 + v^2)\, u_x = 0$

$C^2 u_x = 0 \quad \Rightarrow \quad u_x = 0$

Similarly we can prove that

$u_y = v_x = v_y = 0$

f(z) is analytic

$f'(z) = U_x + iV_x = 0$

As derivative of constant function is 0

Hence f(z) is constant.

Q6] c) Fit a circle of the form y = ax^b to the following data (8)

X	1	2	3	4
y	2.5	8	19	50

Solution:-

Taking log on both sides of $y = ax^b$

$\log y = \log a + b\log x$

let $\log y = Y, \log a = A; b = B; \log x = X$

x	y	X = log x	Y = log y	XY	X^2
1	2.5	0	0.3979	0	0
2	8	0.3010	0.9031	0.2718	0.0906
3	19	0.4771	1.2788	0.6101	0.2276
4	50	0.6020	1.6990	1.0228	0.3624
∑N = 4		1.3801	4.2788	1.9047	0.6806

Putting values in above equations:

4.2788 = 4A + B(1.3801)

1.9047 = (1.3801)A + B(0.6806)

Solving simultaneously;

$A = 0.3466$; $B = 2.09$

$A = \log a \Rightarrow a = 2.22$

$y = 2.22x^{2.09}$
