COMPUTER ENGINEERING

APPLIED MATHS – 3

(CBCGS DEC 2017)

Q1.a) Find the Laplace transform of $\frac{1}{t}e^{-t}\sin t$. **(5)**

Sol: To find : $L\left[\frac{1}{t}e^{-t}\sin t\right]$

$\Rightarrow L[\sin t] = \frac{1}{s^2+1}$ [Since $L\{\sin at\} = \frac{1}{s^2+a^2}$]

By First Shifting Theorem,

$\Rightarrow L[e^{-t}\sin t] = \frac{1}{(s+1)^2+1}$ [Since $L\{e^{at}f(t)\} = \Phi(s-a)$]

$\Rightarrow L\left[\frac{1}{t}e^{-t}\sin t\right] = \int_s^\infty \frac{1}{(s+1)^2+1}ds$ [Effect of division by t]

$\Rightarrow \left[\tan^{-1}(s+1)\right]_s^\infty$ [$\int\frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$]

$\Rightarrow \left[\tan^{-1}(\infty)-\tan^{-1}(s+1)\right]$

$\Rightarrow \frac{\pi}{2} - \tan^{-1}(s+1)$

$\Rightarrow \cot^{-1}(s+1)$ [$\frac{\pi}{2} - \tan^{-1}x = \cot^{-1}x$]

Ans : $L\left[\frac{1}{t}e^{-t}\sin t\right] = \cot^{-1}(s+1)$

Q1.b) Find the inverse Laplace transform of $\frac{1}{\sqrt{2s+1}}$. **(5)**

Sol: To find : $L^{-1}\left[\frac{1}{\sqrt{2s+1}}\right]$

$\Rightarrow L^{-1}\left[\frac{1}{\sqrt{2s+1}}\right] = L^{-1}\left[\frac{1}{2\sqrt{s+\frac{1}{2}}}\right]$

$$\Rightarrow \frac{e^{-\frac{t}{2}}}{\sqrt{2}} \; L^{-1}\left[\frac{1}{\sqrt{s}}\right]$$ [$L^{-1}\{\phi(s+a)\} = e^{-at}L^{-1}[\phi(s)]$]

$$\Rightarrow \frac{e^{-\frac{t}{2}}}{\sqrt{2}} \left[\frac{t^{-\frac{1}{2}}}{\Gamma\frac{1}{2}}\right]$$ [$L^{-1}\left[\frac{1}{s^n}\right] = \frac{t^{n-1}}{\Gamma n}$]

$$\Rightarrow \frac{e^{-\frac{t}{2}}}{\sqrt{2\pi}} \left[t^{-\frac{1}{2}}\right]$$

Ans : $L^{-1}\left[\frac{1}{\sqrt{2s+1}}\right] = \frac{e^{-\frac{t}{2}}}{\sqrt{2\pi}} t^{-\frac{1}{2}}$

Q1.c) Show that the function, f(z) = $\sinh(z)$ is analytic and find $f'(z)$ in terms of z **(5)**

Sol: Given : f(z) = sinh (z)

$\Rightarrow \sinh(x+iy) = \sinh(x)\cosh(iy) + \cosh(x)\sinh(iy)$

$\Rightarrow \sinh(x)\cos(y) + i\cosh(x)\sin(y)$ [cosh(iy)=cos y, sinh(iy)=isin(y)]

Comparing real and imaginary parts,

$u=\sinh(x)\cos(y); v = \cosh(x)\sin(y)$

Differentiating u and v partially with respect to x and y,

$u_x = \cosh(x)\cos(y); u_y = -\sinh(x)\sin(y)$

$v_x = \sinh(x)\sin(y); v_y = \cosh(x)\cos(y)$

From above equations clearly, we can see that : $u_x = v_y$ & $u_y = -v_x$

Thus CR equations are satisfied and thus the function is analytic.

Therefore; $f'(z) = u_x + iv_x$

$f'(z) = \cosh(x)\cos(y) + i\sinh(x)\sin(y)$

$f'(z) = \cos h(x+iy)$

$f'(z) = \cosh(z)$

Ans : $f'(z)$ **= cosh(z)**

Q1.d) Find the Fourier series for f(x) = x in (0,2π). **(5)**

Sol: $f(x) = x$

Fourier series is given by : $f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos n\pi x + b_n \sin n\pi x\right)$

Calculating a_0,

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx \quad => \quad a_0 = \frac{1}{2\pi}\int_0^{2\pi} x dx$$

$$a_0 = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi} dx \quad => \quad a_0 = \frac{1}{2\pi}\left[\frac{4\pi^2}{2} - 0\right] dx$$

$$a_0 = \pi \qquad \text{-----------1}$$

Calculating a_n,

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x\cos(nx)dx$$

$$=> \quad a_n = \frac{1}{\pi}\left[\frac{x\sin(nx)}{n} - \left(\frac{-\cos nx}{n^2}\right)\right]_0^{2\pi} dx$$

$$=> \quad a_n = \frac{1}{\pi}\left[\left\{\frac{2\pi \sin(2n\pi)}{n} + \frac{\cos(2n\pi)}{n^2}\right\} - \left\{0 + \frac{\cos 0}{n^2}\right\}\right]$$

$$=> \quad a_n = \frac{1}{\pi}\left[\left\{0 + \frac{1}{n^2}\right\} - \left\{0 + \frac{1}{n^2}\right\}\right] \quad => a_n = 0 \qquad \text{-----------2}$$

Calculating b_n,

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x\sin(nx)dx$$

$$=> \quad b_n = \frac{1}{\pi}\left[\frac{x(-\cos nx)}{n} - \left(\frac{-\sin nx}{n^2}\right)\right]_0^{2\pi} dx$$

$$=> \quad b_n = \frac{1}{\pi}\left[\left\{\frac{2\pi(-\cos 2n\pi)}{n} + \frac{\sin(2n\pi)}{n^2}\right\} - \left\{0 + \frac{\sin 0}{n^2}\right\}\right]$$

$$=> \quad b_n = \frac{1}{\pi}\left[\left\{\frac{-2\pi}{n} + 0\right\} - \{0+0\}\right] \quad => b_n = \frac{-2}{n} \qquad \text{-----------3}$$

Substituting in the Fourier Series, we get;

$$x = \pi + 0 + \sum_{n=1}^{\infty}\left(\frac{-2}{n}\sin n\pi x\right)$$

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Ans : $x = \pi - 2\sum_{n=1}^{\infty}\left(\frac{\sin n\pi x}{n}\right)$

Q2.a) Use Laplace transform to prove : $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt = \frac{1}{4}\log 5$. **(6)**

Sol: To prove $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt = \frac{1}{4}\log 5$

LHS : $\int_0^\infty e^{-t}\frac{\sin^2 t}{t}dt$

$\Rightarrow L[\sin^2 t] = L\left[\frac{1-\cos(2t)}{2}\right]$

$\Rightarrow \frac{1}{2}L[1 - \cos(2t)]$

$\Rightarrow \frac{1}{2}L\left[\frac{1}{s} - \frac{2}{s^2+4}\right]$ $\left[\ L\{\cos at\} = \frac{s}{s^2+a^2}\ ; L[1] = \frac{1}{s}\ \right]$

$L\left[\frac{\sin^2 t}{t}\right] = \frac{1}{2}\left[\int_s^\infty \frac{1}{s} - \frac{s}{s^2+4}\right]ds$

$= \frac{1}{2}\left[\log s - \frac{1}{2}\log(s^2+4)\right]$

$= -\frac{1}{4}\left[\log(s^2+4) - \log s^2\right]$

$= -\frac{1}{4}\left[\log\left(\frac{s^2+4}{s^2}\right)\right]_s^\infty$

$= \frac{1}{4}\log\left[\frac{s^2+4}{s^2}\right]$

Therefore , $\int_0^\infty e^{-st}\frac{\sin^2 t}{t}dt = \frac{1}{4}\log\left[\frac{s^2+4}{s^2}\right]$

Putting s =1;

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$$\int_0^{\infty} e^{-t}\frac{\sin^2 t}{t}dt = \frac{1}{4}\log\left[\frac{1^2+4}{1}\right]$$

$$\int_0^{\infty} e^{-st}\frac{\sin^2 t}{t}dt = \frac{1}{4}\log[5]$$

Ans : $\int_0^{\infty} e^{-st}\frac{\sin^2 t}{t}dt = \frac{1}{4}\log[5]$

Q2.b) If $\{f(k)\} = \begin{cases} 4^k, k<0 \\ 3^k, k\geq 0 \end{cases}$ **, find Z{f(k)}.** **(6)**

Sol: By definition,

$$Z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k)\,.\,z^{-k}$$

$$Z\{f(k)\} = \sum_{k=-\infty}^{1} 5^k\,.\,z^{-k} + \sum_{k=0}^{\infty} 3^k\,.\,z^{-k}$$

Put k= -n in 1^{st} series,

$$\Rightarrow Z\{f(k)\} = \sum_{n=1}^{\infty} 5^{-n}\,.\,z^{n} + \sum_{k=0}^{\infty} 3^k\,.\,z^{-k}$$

$$\Rightarrow Z\{f(k)\} = \left[\left(\frac{z}{5}\right)+\left(\frac{z}{5}\right)^2+\left(\frac{z}{5}\right)^3+\ldots\right] + \left[1+\left(\frac{3}{z}\right)+\left(\frac{3}{z}\right)^2+\left(\frac{3}{z}\right)^3+\ldots\right]$$

The above two series are sum of infinite GP terms whose summation is given by,

$S = \frac{a}{1-r}$, where a is 1st term and r is the common ratio between the terms

$$\Rightarrow Z\{f(k)\} = \frac{z}{5}\left[\frac{1}{1-\left(z/5\right)}\right] + \left[\frac{1}{1-\left(3/z\right)}\right]$$

$$\Rightarrow Z\{f(k)\} = \frac{z}{5}\left[\frac{5}{5-z}\right] + \left[\frac{z}{z-3}\right]$$

$$\Rightarrow Z\{f(k)\} = \left[\frac{z}{5-z}\right] + \left[\frac{z}{z-3}\right]$$

$$\Rightarrow Z\{f(k)\} = \frac{z(z-3)+z(5-z)}{(5-z)(z-3)}$$

$$\Rightarrow Z\{f(k)\} = \frac{2z}{(5-z)(z-3)}$$

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Ans : $Z\{f(k)\} = \dfrac{2z}{(5-z)(z-3)}$

Q2.c) Show that the function u = $\cos x \cosh y$ is a harmonic function . Find its harmonic conjugate and corresponding analytic function (8)

Sol : Given : $u = \cos x \cosh y$

$u_x = -\sin x \cosh y \quad ; u_y = \cos y \sinh y$

$u_x^2 = -\cos x \cosh y \quad ; u_y^2 = \cos x \cosh y$

From the above equations,

$u_x^2 + u_y^2 = 0$ Thus the Laplace equation is satisfied.

Therefore, u is harmonic

Let $u_x = \Psi_1(x,y)$ and $u_y = \Psi_2(x,y)$

$\Psi_1(z,0) = -\sin z$ and $\Psi_2(z,0) = 0$

By Milne Thompson method,

$$f(z) = \int \Psi_1(z,0)dz - \int \Psi_2(z,0)dz$$

$$f(z) = \int -\sin z dz - \int 0 dz$$

$f(z) = \cos z + c$ This is the required analytic function.

Separating real and imaginary parts, putting z=x+iy,

$f(z) = \cos(x+iy)$

$f(z) = \cos x \cos iy - \sin x \sin iy$

$f(z) = \cos x \cos hy - i \sin x \sinh y$ [cos(iy)=cosh(y) and sin(iy)=isinh(y)]

Therefore, $v = -\sin x \sinh y$

Ans : Required analytic function is $f(z) = \cos z + c$

Harmonic conjugate of u = $v = -\sin x \sinh y$

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Q3.a) Find the equation of the line of regression of Y on X for the following data. (6)

X	5	6	7	8	9	10	11
Y	11	14	14	15	12	17	16

Sol. The Line of regression Y on X is given as y=a + bx.

x	x^2	y	y^2	xy
5	25	11	121	55
6	36	14	196	84
7	49	14	196	98
8	64	15	225	120
9	81	12	144	108
10	100	17	324	170
11	121	16	256	176
$\Sigma = 56$	$\Sigma = 476$	$\Sigma = 99$	$\Sigma = 1427$	$\Sigma = 811$

Here N=7,

The normal equation are given as follows;

$\Sigma y = Na + b\Sigma x$

$\Sigma xy = a\Sigma x + b\Sigma x^2$

Substituting the values from the above table;

$7a + 56b = 99$

$56a + 476b = 811$

Solving the above two equations simultaneously, we get; a=8.714 and b=0.6786

Thus, the equation of line of regression is : 8.714 $+ \ 0.6786x$

Q3.b) Find the bilinear transformation which maps the points 1, -i, 2 on z plane onto 0, 2, -i respectively of w-plane. (6)

Sol: Let the transformation be $w = \frac{az+b}{cz+d}$ ----- i

Putting the given values, $0 = \frac{a+b}{c+d}$; $2 = \frac{-ai+b}{-ci+d}$; $-i = \frac{2a+b}{2c+d}$

From these equations we get, $a + b = 0$ ----- ii

$(a-2c)i + (2d-b) = 0$ ----- iii

$(2c+d)i + (2a+b) = 0$ ----- iv

From ii we get b = -a.

Putting this value of b in iii and iv, we get

$(a-2c)i + (2d+a) = 0$ ----- v

$(2c+d)i + (a) = 0$ ----- vi

Adding v and vi we get

$(a+d)i + 2(a+d) = 0$ Therefore, $(a+d)(i + 2) = 0$

Thus , d = -a [Since, i ≠ -2]

Putting these values of d and b in $2 = \frac{-ai+b}{-ci+d}$, we get $2 = \frac{-ai-a}{-ci-a} = \frac{a(1+i)}{ci+a}$

Therefore, $2ci + 2a = a + ai$ $\Rightarrow 2ci = -a + ai$

$\Rightarrow 2ci = ai^2 + ai$ $\Rightarrow 2ci = ai(i + 1)$

$2c = a(1 + i)$ $\Rightarrow c = \left(\frac{1+i}{2}\right)a$

Putting these values o b, c, d in (i),

$$w = \frac{az-a}{\left(\frac{1+i}{2}\right)az-a}$$

$$w = \frac{z-1}{\left(\frac{1+i}{2}\right)z-1}$$

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$$w = \frac{2(z-1)}{(1+i)z-2}$$

Ans : $w = \dfrac{2(z-1)}{(1+i)z-2}$

Q3.c) Find half range sine series for $f(x) = \begin{cases} x & ,0<x<\frac{\pi}{2} \\ \pi-x & ,\frac{\pi}{2}<x<\pi \end{cases}$. **(8)**

Hence find the sum of $\sum_{(2n-1)}^{\infty}\frac{1}{n^4}$.

Sol: Half range sine series is given by:

$$f(x) = \sum b_n \sin nx$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx\,dx$$

$$\Rightarrow \frac{2}{\pi}\left[\int_0^{\pi/2} x\sin nx\,dx + \int_{\pi/2}^{2\pi}(\pi-x)\sin nx\,dx\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\left\{x\left(\frac{-\cos nx}{n}\right)-\left(\frac{-\sin nx}{n^2}\right)(1)\right\}_0^{\pi/2}+\left\{(\pi-x)\left(\frac{-\cos nx}{n}\right)-\left(\frac{-\sin nx}{n^2}\right)(-1)\right\}_{\pi/2}^{\pi}\right]$$

$$\Rightarrow \frac{2}{\pi}\left[\left\{\frac{\pi}{2}\frac{\cos(n\pi/2)}{n}+\frac{\sin(n\pi/2)}{n^2}-0-0\right\}+\left\{0-0+\frac{\pi}{2}\frac{\cos(n\pi/2)}{n}+\frac{\sin(n\pi/2)}{n^2}\right\}\right]$$

$$\Rightarrow \frac{4}{\pi}\frac{\sin(n\pi/2)}{n^2}$$

$$b_1 = \frac{4}{\pi}\frac{1}{1^2};b_2 = 0;b_3 = -\frac{4}{\pi}\frac{1}{3^2};b_4 = 0;\ldots.$$

$$f(x) = \frac{4}{\pi}\left[\frac{\sin x}{1^2}-\frac{\sin 3x}{3^2}+\frac{\sin 5x}{5^2}-\frac{\sin 7x}{7^2}+\ldots\right]$$

By Parseval's identity;

$$\frac{1}{\pi}\int_0^{\pi}[f(x)]^2dx = \frac{1}{2}\left[b_1^{\,2}+b_2^{\,2}+b_3^{\,3}+b_4^{\,2}+\ldots+\infty\right] \qquad \text{----- i}$$

$$\frac{1}{\pi}\left[\int_0^{\pi/2}x^2dx+\int_{\pi/2}^{\pi}(\pi-x)^2dx\right] = \frac{1}{2}\left[b_1^{\,2}+b_2^{\,2}+b_3^{\,3}+b_4^{\,2}+\ldots+\infty\right]$$

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LHS : $\frac{1}{\pi}\left[\int_0^{\pi/2} x^2 dx+\int_{\pi/2}^{\pi}(\pi^2-2\pi x+x^2)dx\right]$

$$=\frac{1}{\pi}\left[\left\{\frac{x^3}{3}\right\}_0^{\pi/2}+\left\{\pi^2 x-\pi x^2+\frac{x^3}{3}\right\}_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\left\{\frac{x^3}{24}-0\right\}+\left\{\pi^3-\pi^3+\frac{\pi^3}{3}\right\}-\left\{\frac{\pi^3}{2}-\frac{\pi^3}{4}+\frac{\pi^3}{24}\right\}\right]$$

$$= \frac{\pi^2}{12}$$

$$\frac{\pi^2}{12} = \frac{1}{2}\left[\frac{16}{\pi^2}.\frac{1}{1^4}+\frac{16}{\pi^2}.\frac{1}{3^4}+\frac{16}{\pi^2}.\frac{1}{5^4}+...\right]$$

$$\Rightarrow \frac{\pi^2}{96} = \left[\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+...\right]$$

$$\Rightarrow \sum_{(2n-1)}^{\infty}\frac{1}{n^4}=\frac{\pi^2}{96}, n = 1,2,3,...$$

Ans : $\sum_{(2n-1)}^{\infty}\frac{1}{n^4}=\frac{\pi^2}{96}$**,** $n = 1,2,3,...$

Q4.a) Find the inverse Laplace Transform using convolution theorem

$\frac{1}{(s-a)(s+a)^2}$ **(6)**

Sol: $\phi_1(s) = \frac{1}{s-a}$; $\phi_2(s) = \frac{1}{(s+a)^2}$

$$L^{-1}[\phi_1(s)] = L^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$L^{-1}[\phi_2(s)] = L^{-1}\left[\frac{1}{(s+a)^2}\right] = e^{-at}t$$

$$L^{-1}[\phi(s)] = \int_0^t e^{au}.e^{-a(t-u)}(t-u)du$$

$$= \int_0^t e^{au}.e^{-a(t-u)}(t-u)du$$

$$= e^{-at}\int_0^t e^{2au}(t-u)du$$

$$= e^{-at}\left[(t-u)\frac{e^{2au}}{2a}-\frac{e^{2au}}{4a^2}(-1)\right]_0^t$$

$$= e^{-at}\left[0+\frac{e^{2at}}{4a^2}-\left\{\frac{t}{2a}+\frac{1}{4a^2}\right\}\right]$$

$$= \frac{1}{4a^2}[e^{at}-2ate^{-at}+e^{-at}]$$

Ans : $L^{-1}\left[\frac{1}{(s-a)(s+a)^2}\right] = \frac{1}{4a^2}[e^{at}-2ate^{-at}+e^{-at}]$

Q4.b) Calculate the coefficient of correlation between X and Y from the following data (6)

X	**8**	**8**	**7**	**5**	**6**	**2**
Y	**3**	**4**	**10**	**13**	**22**	**8**

Sol:

x	x^2	y	y^2	xy
8	64	3	9	24
8	64	4	16	32
7	49	10	100	70
5	25	12	144	65
6	36	22	484	132
2	4	8	64	16
Σ 36	Σ 242	Σ 60	Σ 842	Σ 339

Here N=6,

$\bar{X} = \frac{36}{6} = 6$ and $\bar{Y} = \frac{60}{6} = 10$

Coefficient of correlation ,

$$r = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\sqrt{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}\sqrt{y^2 - \frac{(\Sigma y)^2}{n}}}$$

Substituing the values , we get :

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$$r = \frac{339 - \frac{36 \times 60}{6}}{\sqrt{242 - \frac{(36)^2}{6}}\sqrt{842 - \frac{(60)^2}{6}}}$$

$r = -0.2647$

Ans : Coefficient of correlation, r = -0.2647

Q4.c) Find the inverse Z-transform of : **(8)**

i) $\frac{1}{(z-a)^2}$**,** $|z|$ **< a**

ii) $\frac{1}{(z-3)(z-2)}$**,** $|z|$ **> 3**

Sol: i) $F(z) = \frac{1}{(z-a)^2}, |z| < a$

$$\frac{1}{a^2[1-\left(\frac{z}{a}\right)]^2} \Rightarrow \frac{1}{a^2}\left(1-\frac{z}{a}\right)^{-2}$$

$$\Rightarrow \frac{1}{a^2}[1 + 2\left(\frac{z}{a}\right)^1 + 3\left(\frac{z}{a}\right)^2 + 4\left(\frac{z}{a}\right)^3 + \ldots + (n+1)\left(\frac{z}{a}\right)^n]$$

Coefficient of $z^n = \frac{n+1}{a^{n+2}}$; $n \geq 0$

Put $n = -k$, $z^{-k} = \frac{-k+1}{a^{-k+2}}$; $k \leq 0$

ii) $F(z) = \frac{1}{(z-3)(z-2)}, |z| > 3$

$$\frac{1}{(z-3)(z-2)} = \frac{A}{z-3} + \frac{B}{z-2}$$

$1 = A(z-2) + B(z-3)$

Putting z=2; 1 = -B => B = -1

Putting z=3; 1 = A => A = 1

$$\frac{1}{(z-3)(z-2)} = \frac{1}{z-3} - \frac{1}{z-2}$$

RHS

$$\Rightarrow \frac{1}{z(1-\frac{3}{z})} - \frac{1}{z(1-\frac{2}{z})}$$

$$\Rightarrow \frac{1}{z}\left[1-\frac{3}{z}\right]^{-1} - \frac{1}{z}\left[1-\frac{2}{z}\right]^{-1}$$

$$\Rightarrow \frac{1}{z}[1 + \frac{3}{z} + \left(\frac{3}{z}\right) + \left(\frac{3}{z}\right) + \ldots + \left(\frac{3}{z}\right)^{k-1}] - \frac{1}{z}\left[1+\frac{2}{z}+\left(\frac{2}{z}\right)+\left(\frac{2}{z}\right)+\ldots+\left(\frac{2}{z}\right)^{k-1}\right]$$

Coefficient of z^{-k} = $3^{k-1} - 2^{k-1}$; $k \geq 1$

$$Z^{-1}[F(z)] = 3^{k-1} - 2^{k-1}$$

Q5.a) Using Laplace transform evaluate $\int_0^{\infty} e^{-t}\left(1+2t-t^2+t^3\right)H(t-1)dt$ (6)

Sol : To evaluate $\int_0^{\infty} e^{-t}\left(1+2t-t^2+t^3\right)H(t-1)dt$

$\Rightarrow f(t) = 1 + 2t - t^2 + t^3$; a=1

$\Rightarrow f(t + 1) = 1 + 2(t + 1) - (t+1)^2 + (t+1)^3$

$= 1 + 2t + 2 - (t^2 + 2t + 1) + t^3 + 3t^2 + 3t + 1$

$= t^3 + 2t^2 + 3t + 3$

$L[f(t+1)] = L[t^3 + 2t^2 + 3t + 3]$

$$= \frac{3!}{s^4} + 2\frac{2!}{s^3} + \frac{3}{s^2} + \frac{3}{s} \quad \text{----- i}$$

We know, $L[f(t)H(t-a)] = e^{-as}L[f(t + a)]$

Substituting the value of L[f(t+a)] in above equation, we get

$$L\left[\left(1+2t-t^2+t^3\right)H(t-1)\right] = e^{-as}\left[\frac{3!}{s^4}+2\frac{2!}{s^3}+\frac{3}{s^2}+\frac{3}{s}\right]$$

$$\int_0^{\infty} e^{-st}\left(1+2t-t^2+t^3\right)H(t-1)dt = e^{-s}\left[\frac{3!}{s^4}+2\frac{2!}{s^3}+\frac{3}{s^2}+\frac{3}{s}\right]$$

Putting s=1 in the above equation;

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$$\int_0^\infty e^{-t}\left(1+2t-t^2+t^3\right)H(t-1)dt = e^{-1}\left[\frac{3!}{1^4}+2\frac{2!}{1^3}+\frac{3}{1^2}+\frac{3}{1}\right]$$

$$=e^{-1}[6+4+3+3] = \frac{16}{e}$$

Ans :

$$\int_0^\infty e^{-t}\left(1+2t-t^2+t^3\right)H(t-1)dt = \left[\frac{16}{e}\right]$$

Q5.b) Show that the set of functions cos x, cos 2x, cos 3x, … is a set of orthogonal functions over [-π,π]. Hence construct set of orthonormal functions. (6)

Sol : We have $f_n(x) = \cos nx$;n=1, 2, 3, …

Therefore, $\int_{-\pi}^{\pi} f_m(x) \cdot f_m(x)dx \Rightarrow \int_{-\pi}^{\pi}\cos mx.\cos nxdx$

$$\Rightarrow \frac{1}{2}\int_{-\pi}^{\pi}\cos (m+n)x+\cos(m-n)xdx$$

$$\Rightarrow \frac{1}{2}\left[\frac{\sin (m+n)x}{m+n}+\frac{\sin (m-n)x}{m-n}\right]_{-\pi}^{\pi}$$

Now two cases arise:

i. When m ≠ n :

$$= \frac{1}{2}\left[\left\{\frac{\sin (m+n)\pi}{m+n}+\frac{\sin (m-n)\pi}{m-n}\right\}-\left\{\frac{-\sin (m+n)\pi}{m+n}-\frac{\sin (m-n)\pi}{m-n}\right\}\right]$$

$$= \left[\left\{\frac{\sin (m+n)\pi}{m+n}+\frac{\sin (m-n)\pi}{m-n}\right\}\right]$$

$$= 0$$

ii. When m=n:

$$\int_{-\pi}^{\pi}\cos^2 nxdx = \int_{-\pi}^{\pi}\frac{1+\cos 2x}{2}dx$$

$$\Rightarrow \frac{1}{2}\left[x+\frac{\sin2x}{2}\right]_{-\pi}^{\pi}$$

$$\Rightarrow \frac{1}{2}[\pi+0-(-\pi+0)] \Rightarrow \pi \neq 0$$

Therefore the functions are orthogonal in [-π,π]

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$$\int_{-\pi}^{\pi}[f(x)]^2dx = \pi$$

dividing the above equation by π;

$$\Rightarrow \frac{1}{\pi}\int_{-\pi}^{\pi}[f(x)]^2dx = 1$$

$$\Rightarrow \int_{-\pi}^{\pi}\frac{1}{\sqrt{\pi}}f(x).\frac{1}{\sqrt{\pi}}f(x)dx = 1$$

This is obviously an orthonormal set where $\phi(x) = \frac{1}{\sqrt{\pi}}\cos nx$

Thus the required orthonormal set is $\frac{1}{\sqrt{\pi}}\cos x, \frac{1}{\sqrt{\pi}}\cos 2x, \frac{1}{\sqrt{\pi}}\cos 3x, \ldots$

Q5.c) Solve using Laplace transform: (8)

$(D^3 - 2D^3 + 5D)y = 0$ with $y(0) = 0; y'(0) = 0; y''(0) = 1$

Sol: Let $L(y) = \dot{y}$

Taking Laplace transform on both sides of the given equation ;

$$L(y''') - 2L(y'') + 5L(y') = 0$$

$$\Rightarrow L(y') = s(\dot{y}) - y(0)\ ;\ L(y'') = s^2\dot{y} - sy(0) - y'(0)\ ;\ L(y''') = s^3\dot{y} - s^2y(0) - sy'(0) - y''(0)$$

From the given conditions;

$$L(y') = s(\dot{y})\ ;;\ L(y'') = s^2\dot{y}\ ;;\ L(y''') = s^3\dot{y} - 1$$

Therefore the equation becomes;

$$\Rightarrow s^3\dot{y} - 1 - 2s^2\dot{y} + 5s(\dot{y}) = 0$$

$$\Rightarrow \dot{y} = \frac{1}{s^3-2s^2+5s}$$

Taking inverse Laplace transform,

$$\Rightarrow y = L^{-1}\left[\frac{1}{s^3-2s^2+5s}\right]$$

$$\Rightarrow y = L^{-1}\left[\frac{1}{s(s^2-2s+5)}\right] \Rightarrow L^{-1}\left[\frac{1}{s[(s-1)^2+2^2]}\right]$$

We obtain the inverse by convolution theorem,

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$\phi_1(s) = \frac{1}{(s-1)^2+2^2}$; $\phi_2(s) = \frac{1}{s}$

$f_1(t) = L^{-1}[\phi_1(s)] \Rightarrow L^{-1}\left(\frac{1}{(s-1)^2+2^2}\right) \Rightarrow e^t L^{-1}\left(\frac{1}{(s)^2+2^2}\right) = \frac{1}{2}.e^t.\sin 2t$

$f_2(t) = L^{-1}[\phi_2(s)] \Rightarrow L^{-1}[\frac{1}{s}] \Rightarrow 1$

$\Rightarrow f_1(u) = \frac{1}{2}.e^u.\sin 2u$

$\Rightarrow L^{-1}[\phi(s)] = \frac{1}{2}\left[\frac{1}{1+2^2}[e^u(\sin 2u-2\cos 2u)]\right]_0^t$

*The above integral is of this format : $[\int e^{ax}\sin bx = \frac{1}{a^2+b^2}(e^{ax}\{\sin ax-b\cos bx\})]$

$\Rightarrow L^{-1}[\phi(s)] = \frac{1}{2}\left[\frac{1}{5}[e^t(\sin 2t-2\cos 2t)+2]\right]$

$\Rightarrow L^{-1}[\phi(s)] = \left[\frac{1}{10}[e^t(\sin 2t-2\cos 2t)+2]\right]$

The solution is : $\left[\frac{1}{10}[e^t(\sin 2t-2\cos 2t)+2]\right]$

Q6.a) Find the Complex Form of the Fourier Series for f(x) = 2x in (0,2π) (6)

Sol : f(x)=2x, range (0, 2π);

$\Rightarrow \sum_{-\infty}^{\infty} C_n e^{inx}$; where $C_n = \frac{1}{2\pi}\int_0^{2\pi} f(x)e^{-inx}dx$

Hence, $C_n = \frac{1}{2\pi}\int_0^{2\pi} 2x.e^{-inx}dx$ ----- i

$\Rightarrow \frac{1}{\pi}\int_0^{2\pi} x.e^{-inx}dx$

$\Rightarrow \frac{1}{\pi}\left[x.\frac{e^{-inx}}{-in} - \frac{e^{-inx}}{(in)^2}\cdot 1\right]_0^{2\pi}$

$\Rightarrow \frac{1}{\pi}\left[-\frac{2\pi e^{-i2n\pi}}{in} + \frac{e^{i2n\pi}}{n^2} - 0 - \frac{1}{n^2}\right]$

$\Rightarrow \frac{1}{\pi}\left[-\frac{2\pi}{in} + \frac{1}{n^2} - 0 - \frac{1}{n^2}\right] \Rightarrow \frac{1}{\pi}(-\frac{2\pi}{in})$

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$\Rightarrow \left(-\frac{2}{in}\right)\left(\frac{i}{i}\right) \Rightarrow \frac{2i}{n}$ {n≠0}

For n=0, substitute it in (i);

$C_0 = \frac{1}{2\pi}\int_0^{2\pi} 2x dx \Rightarrow \frac{1}{\pi}\left(\frac{x^2}{2}\right)_0^{2\pi} = \frac{1}{2\pi}(4\pi^2) = 2\pi$

Therefore, $f(x) = 2\pi + \sum_{-\infty}^{\infty} \frac{2i}{n} e^{inx}$

$\Rightarrow$**$f(x) = 2\pi + 2i\sum_{-\infty}^{\infty} \frac{e^{inx}}{n}$**

Q6.b) If $f(z)$ and $f(z)$ are both analytic, prove that f(z) is constant (6)

Sol: $f(z) = u + iv$

$f(z) = u + i(-v)$

For $f(z)$:

$u_x = v_y$ ----- i

$u_y = -v_x$ ----- ii

For $f(z)$:

$u_x = -v_y$ ----- iii

$u_y = -(-v_x)$ ----- iv

From i and iii;

$v_y = -v_y \Rightarrow 2v_y = 0 \Rightarrow v_y = 0$

From ii and iv;

$v_x = -v_x \Rightarrow 2v_x = 0 \Rightarrow v_x = 0$

Substituting in i and ii,

$u_x = u_y = 0$

Therefore u=k and v=k [partial derivatives of constant are zero]

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Hence u+iv is constant

⇨ f(z) is constant.

Q6.c) Fit a curve of the form y = ab^x to the following data. (8)

X	1	2	3	4	5	6
Y	151	100	61	50	20	8

Sol: $y = ab^x$

Taking log on both sides,

$\log y = \log a + x \log b$

Let log y = Y, log a= A, x=X and log b= B

=>Y=A+X(B)

x	y	X	Y	X^2	XY
1	151	1	2.1789	1	2.1789
2	100	2	2	4	4
3	61	3	1.7853	9	5.3559
4	50	4	1.6989	16	6.7956
5	20	5	1.3010	25	6.5050
6	8	6	0.9031	36	5.4186
		∑21	∑9.8672	∑91	∑30.254

Here , N=6

$\Sigma Y = NA + B\Sigma X$

$\Sigma XY = A\Sigma X + B\Sigma X^2$

Substituting the values from the above table;

$6A + 21B = 9.8672$

21A+91B = 30.254

On solving simultaneously ;

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A=2.5 and B=-0.2446

Hence, b= antilog(-0.2446) =>0.5668 $[10^{-0.2446} = 0.56676]$

a=antilog(2.5) =>316.2278 $[10^{2.5} = 0.56676]$

Therefore , $y = (316.2278)(0.5668)^x$

Ans : y = $(316.2278)(0.5668)^x$

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