Duration: 2 ½ Hrs Marks: 75

N.B. : (1) All questions are compulsory.
(2) Figures to the right indicate marks.

Q 1 (A) Attempt any One of the following: (8)

(i) Prove that the elimination of the arbitrary function ϕ from the equation $\phi(u,v) = 0$, where u and v are functions of x, y and z where z is a function of x and y, gives the partial differential equation $\frac{\partial(u,v)}{\partial(y,z)} * p + \frac{\partial(u,v)}{\partial(z,x)} * q = \frac{\partial(u,v)}{\partial(x,y)}$.

(ii) Explain the following different types of first order partial differential equations. Also give an example for each of the following terms.
(I) Linear partial differential equations.
(II) Semi-linear equations.
(III) Quasi-linear equations.

(B) Attempt any Two of the following: (12)

(i) Let $f(x,y)$ be a homogeneous function of x and y of degree n. Then show that $z = f(x,y)$ satisfies the partial differential equation $x\ p + y\ q = nz$.

(ii) Solve the Lagrange's partial differential equation $(y - zx)\ p + (x + yz)\ q = x^2 + y^2$ by choosing the sets of multipliers as $x, -y, z$ and $-y, -x, 1$ or of your own choice.

(iii) It is given that the equation $yz\ dx + 2xz\ dy - 3xy\ dz = 0$ is integrable. Find its integral.

Q 2 (A) Attempt any One of the following: (8)

(i) Define a compatible system of two first order partial differential equations.
If $p = \phi(x,y,z)$ and $q = \psi(x,y,z)$ are obtained by solving the partial differential equations $f(x,y,z,p,q) = 0$ and $g(x,y,z,p,q) = 0$ for p and q then prove that

$$\psi_x + \phi * \psi_z = \frac{1}{\frac{\partial(f,g)}{\partial(p,q)}} \left[\frac{\partial(f,g)}{\partial(x,p)} + \phi\ \frac{\partial(f,g)}{\partial(z,p)}\right]$$

(ii) State Charpit's Auxiliary equations and explain the Charpit's method to find a complete integral of the given partial differential equation $f(x,y,z,p,q) = 0$.

(B) Attempt any Two of the following: (12)

(i) Show that the partial differential equations $p = x^2 - ay, q = y^2 - ax$ are compatible and solve them.

(ii) For the partial differential equations $xp - yq = x$ and $x^2p + q = xz$, answer the following questions.
(I) Show that $\frac{\partial(f,g)}{\partial(x,p)} + p\ \frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\ \frac{\partial(f,g)}{\partial(z,q)} = 0$.
(II) Solve the given two equations for p and q.
(III) Construct the corresponding Pfaffian differential equation.

(iii) Write the equation $u_x^2 + u_y^2 + u_z = 1$ in the form $f(x, y, z, u_x, u_y, u_z) = 0$ and find the Jacobi's auxiliary equations. Hence find a complete integral of the given equation.

Q 3 (A) Attempt any One of the following: (8)

(i) Write a short note on the characteristic strip and characteristic curve for a non linear first order partial differential equation $f(x, y, z, p, q) = 0$.

(ii) Describe the steps followed in finding the integral surface for the partial differential equation $P(x, y, z)\, p + Q(x, y, z)\, q = R(x, y, z)$ containing the data curve $x = x_0(s)$, $y = y_0(s), z = z_0(s), s \in I$, where $I \subseteq \mathbb{R}$ is an interval and x_0, y_0, z_0 are continuously differentiable functions on I.

(B) Attempt any Two of the following: (12)

(i) Solve the initial value problem for the quasi-linear equation $z\, p + 2q = 2$ containing the initial data curve $x_0(s) = s, y_0(s) = s, z_0(s) = \frac{s}{2}, \quad 0 \leq s \leq 1$.

(ii) Find the initial strips and the characteristic differential equations for the integral surface of $pq = xy$ which passes through the initial data curve $x = x_0(s) = s, y = y_0(s) = 0, z = z_0(s) = s$.

(iii) Solve the equation $-yp + 4xq = z$ with the initial conditions $z(x, 0) = x^2, x \geq 0$.

Q 4 Attempt any Three of the following. (15)

(a) Eliminate the arbitrary function f from $z = e^{ax+by} f(ax - by)$ and obtain the corresponding partial differential equation.

(b) Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is an integral of $z^2(1 + p^2 + q^2) = 1$.

(c) Show that the two partial differential equations $p = x - \frac{y}{x^2 + y^2}, q = y + \frac{x}{x^2 + y^2}$ are compatible.

(d) Prove that a complete integral of $z = px + qy - 2p - 3q$ represents a family of planes. Also show that each member of the family passes through $(2, 3, 0)$.

(e) Write the Characteristic differential equations of the partial differential equation $\frac{1}{2}(p^2 + q^2) + (p - x)(q - y) - z = 0$.

(f) What is an initial strip of a non-linear partial differential equation $f(x, y, z, p, q) = 0$ with initial data curve $x = x_0(s), y = y_0(s), z = z_0(s)$?