Duration:[2½Hours] [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate full marks.

1. (a) Attempt any **ONE** question:
   i. Show that a nontrivial graph is bipartite if and only if it contains no odd cycle. (8)
   ii. State and prove *Havel – Hakimi* theorem for degree sequence of a graph G. (8)
   (b) Attempt any **TWO** questions:
   i. Prove that every (p,q) graph with $q \geq p$ contains a cycle. Is it true if $q \geq p-1$? Justify. (6)
   ii. Show that in a party of 6 or more people, either there are 3 persons who know one another or there are three persons who do not know one another. (6)
   iii. Explain Dijkstra's algorithm and show that Dijkstra's algorithm produces the shortest path. (6)

2. (a) Attempt any **ONE** question:
   i. Define a cut vertex for a simple graph G. Show that vertex v of a simple graph G is a cut vertex if and only if there exists two vertices x and y such that vertex v is on every $x-y$ path in G. (8)
   ii. State and prove Cayley's formula for spanning trees. (8)
   (b) Attempt any **TWO** questions:
   i. Show that every nontrivial graph contains at least two vertices which are non-cut vertices. (6)
   ii. Prove that if G is a connected graph of order $p \geq 3$ and G has a cut edge then G contains a cut vertex. Is the converse true? Justify. (6)
   iii. Show that a graph is connected if and only if it has a spanning tree. (6)

3. (a) Attempt any **ONE** question:
   i. Show that a nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree. (8)
   ii. Prove that the cube graph Q_k is connected bipartite $k-$regular graph with 2^k vertices. (8)
   (b) Attempt any **TWO** questions:
   i. Define closure of a graph $C(G)$. Show that a simple graph is Hamiltonian if and only if its closure is Hamiltonian. (6)
   ii. If G is a (p,q) graph with $p \geq 3$ and $q \geq \frac{1}{2}(p-1)(p-2)+2$, then prove that G is Hamiltonian. (6)
   iii. If G is Hamiltonian graph then for every nonempty proper subset S of $V(G)$, prove that $\omega(G-S) \leq |S|$. (6)

4. Attempt any **THREE** questions:

(a) Let A denote the adjacency matrix of a connected graph G with $V(G) = \{v_1, v_2, \ldots v_n\}$, then show that the distance between v_i and v_j is the smallest integer $n \geq 0$ such that $(A^n)_{ij} \neq 0$. (5)

(b) Find three non-isomorphic graphs with degree sequence 3,3,2,2,2,2. Justify your answer. (5)

(c) Show that any two longest paths in a connected graph G has a vertex in common. (5)

(d) Describe Depth First Search (DFS) algorithm. Use DFS to find spanning tree for the complete graph K_5. (5)

(e) Describe Fluery's Algorithm to find a closed Eulerian trail. (5)

(f) Show that the line graph a simple graph G is a path if and only if G is a path. (5)

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