SE/III/R19/MECH/FH-2022/EM-III/03-06-2022

University Of Mumbai
Examination Summer 2022

QP Code: 90787

Time: 2.30 hours | Max. Marks: 80

Q1.	**Choose the correct option for following questions. All the Questions are compulsory and carry equal marks**
1.	What is the Laplace transform of $\int_0^t \sin 5u \, du$?
Option A:	$\frac{5}{s(s^2+25)}$
Option B:	$\frac{5}{s(s^2-25)}$
Option C:	$\frac{1}{s(s^2-25)}$
Option D:	$\frac{1}{s^2+25}$
2.	Find value of b_n in the Fourier expansion of function $f(x)=(2-x^2)$ in the interval $(0,2)$.
Option A:	$\frac{2}{n\pi}+\frac{2}{n^3\pi^3}$
Option B:	$\frac{2}{n\pi}$
Option C:	$\frac{4}{n\pi}$
Option D:	$\frac{4}{n^3\pi^3}$
3.	If $f(z)=e^z$ is an analytic function, then real part is given by
Option A:	$e^x \cos y$
Option B:	$\cos y$
Option C:	$-e^x \sin y$
Option D:	$\sin y$
4.	$L^{-1}[1/(S+2)^4]$
Option A:	$e^{-2t}.t^3/3$
Option B:	$e^{-2t}.t^4/6$
Option C:	$e^{-3t}.t^3/6$
Option D:	$e^{-2t}.t^3/6$
5.	If $f(x) = \cos x$ defined in $(-\pi,\pi)$then the value Fourier coefficient b_n is ,
Option A:	0
Option B:	π
Option C:	$\frac{\pi}{(n^2-1)}$

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Option D:	$\frac{2\pi}{(n^2-1)}[(-1)^n - 1]$
6.	A function $u(x,y)$ is harmonic if and only if,
Option A:	$u_{xx} + u_{yy} = 0$
Option B:	$u_x + u_y = 0$
Option C:	$u_{xy} + u_{yx} = 0$
Option D:	$u_x - u_y = 0$
7.	Find $L^{-1}\left[\frac{3s+4}{s^2+16}\right]$
Option A:	$4.\sin 4t + \cos 4t$
Option B:	$\cos 4t + \sin 3t$
Option C:	$3.\cos 4t + \sin 4t$
Option D:	$\sin 3t + \cos 4t$
8.	If characteristic equation of matrix A of order 3×3 is $\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$. Then by Cayley Hamilton theorem A^{-1} is equal to
Option A:	$A^3 - 3A^2 + 3A - I$
Option B:	$A^2 - 3A - 3I$
Option C:	$3A^2 - 3A - I$
Option D:	$A^2 - 3A + 3I$
9.	The Laplace Transform of $t.e^{at}$
Option A:	$\frac{1}{s}$
Option B:	$\frac{1}{(s-a)^2}$
Option C:	$\frac{1}{(s+a)^2}$
Option D:	$\frac{1}{s^2}$
10.	The equation of one dimensional heat flow is given by
Option A:	$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$
Option B:	$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$
Option C:	$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
Option D:	$\frac{\partial u}{\partial t} = c^2\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right)$

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Q2 **(20 Marks)**	**Solve any Four out of Six5 marks each**
A	Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$ by Bender-Schmidt method, given $u(0,t) = 0$, $u(5,t) = 0$, $u(x,0) = x^2(25 - x^2)$ Assume h=1 & find the values of u up to t=3
B	Using convolution theorem find inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+4)}$
C	Find the Laplace transform of $\cos t.\cos 2t.\cos 3t$
D	Using Cayley-Hamilton theorem, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.
E	Find k such that $\frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{kx}{y}\right)$ is analytic.
F	Find Fourier expansion of $f(x) = x^2$ in the interval $(0, 2\pi)$.

Q3 **(20 Marks)**	**Solve any Four out of Six5 marks each**
A	Find $L^{-1}\{\frac{s-2}{(s^2+4s+8)}\}$
B	Find Half Range Cosine Series for f(x)=x; 0<x<2
C	Find the orthogonal trajectories of the curve is $e^x \cos y - xy = c$
D	Solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$, under the conditions $u(0,t) = 0;\ u(1,t) = t, u(x,0) = 0$ $h=\frac{1}{4}$ (one –time step) using Crank Nicholson's method
E	Show that $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalizable. Determine transforming and diagonal matrix.
F	Find L.T. of the following functions:- (i) $te^{-4t}\sin 3t$ (ii) $\frac{1}{t}[\cos(2t) - \cos(3t)]$

Q4 **(20 Marks)**	**Solve any Four out of Six5 marks each**
A	Evaluate $\int_0^\infty e^t \sin 2t \cos 3t\ dt$
B	Find Fourier series of $f(x) = x^2$ in the interval $(-\pi, \pi)$. Hence prove that $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \ldots\ldots = \frac{\pi^2}{6}$
C	An elastic string stretched between the fixed points (0, 0) and (1, 0) initially in the position $y = A\sin(\pi x)$ and released from rest. Find the displacement y(x,t)
D	If $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$ Calculate e^A and 5^A
E	Find an analytic function $f(z)$ whose imaginary part is

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	$e^{-x}\left(y\sin y + x\cos y\right)$
F	Find the inverse Laplace transform of $F(s) = \log\left(\dfrac{s^2 + a^2}{\sqrt{s+b}}\right)$.

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