IT & COMP Sem-III (CBGS) 12/5/16

App. Maths - III

QP Code : 30557

(3 Hours) **[Total Marks : 80**

1) Question No. **1** is compulsory.

2) Attempt any **THREE** of the remaining.

3) **Figures** to the **right** indicate **full** marks.

Q 1.A) If $\int_0^\infty e^{-2t}\sin(t+\alpha)\cos(t-\alpha)dt = \frac{1}{4}$, find α (5)

B) Find half range Fourier cosine series for $f(x) = x,\ \ 0 < x < 2$ (5)

C) If $u(x,y)$ is a harmonic function then prove that $f(z) = u_x - iu_y$ is an analytic function. (5)

D) Prove that $\nabla f(r) = f'(r)\frac{\bar{r}}{r}$ (5)

Q.2) A) If $v = e^x \sin y$, prove that v is a harmonic function. Also find the corresponding analytic function. (6)

B) Find Z-transform of $f(k) = b^k, k \geq 0$ (6)

C) Obtain Fourier series for $f(x) = \frac{3x^2 - 6x\pi + 2\pi^2}{12}$ in $(0, 2\pi)$,

where $f(x+2\pi) = f(x)$. Hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$ (8)

Q.3) A) Find inverse Laplace of $\frac{(s+3)^2}{(s^2+6s+5)^2}$ using Convolution theorem (6)

B) Show that the set of functions {sinx, sin3x, sin5x,.....} is orthogonal over $[0, \pi/2]$. Hence construct orthonormal set of functions (6)

C) Verify Green's theorem for $\int_c \frac{1}{y}dx + \frac{1}{x}dy$ where C is the boundary of region defined by $x = 1, x = 4, y = 1$ and $y = \sqrt{x}$ (8)

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Q.4) Find $Z\{k^2 a^{k-1} U(k-1)\}$ (6)

B) Show that the map of the real axis of the z- plane is a circle under the transformation $w = \frac{2}{z+i}$. Find its centre and the radius. (6)

C) Express the function $f(x) = \begin{cases} \sin x & |x| < \pi \\ 0 & |x| > \pi \end{cases}$ as Fourier sine Integral. (8)

Q.5) A) Using Gauss Divergence theorem evaluate $\iint_s \bar{N}.\bar{F}\,ds$ where $\bar{F} = x^2 i + zj + yzk$ and S is the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1 (6)

B) Find inverse Z-transform of $F(z) = \frac{z}{(z-1)(z-2)}$, $|z| > 2$ (6)

C) Solve $(D^2+3D+2)y = e^{-2t}\sin t$, with $y(0) = 0$ and $y'(0) = 0$ (8)

Q.6) A) Find Fourier expansion of $f(x) = 4 - x^2$ in the interval (0,2) (6)

B) A vector field is given by $\bar{F} = (x^2 + xy^2)\, i + (y^2 + x^2 y)\, j$. Show that $\bar{F}$ is irrotational and find its scalar potential. (6)

C) Find (i) $L^{-1}\left\{\tan^{-1}\left(\frac{a}{s}\right)\right\}$

(ii) $L^{-1}\left(\frac{e^{-\pi s}}{s^2-2s+2}\right)$ (8)

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