S.E. Sem III (CBGS) (Computer & I.T.). 20/11/15

A.M.-III

Q.P. Code : 5067

(3 Hours) **[Total Marks : 80**

Instructions:

1) Question No. **1** is compulsory.

2) Attempt any **THREE** of the remaining.

3) **Figures** to the **right** indicate **full** marks.

Q 1. A) Find Laplace of $\{t^5 \cosh t\}$ (5)

B) Find Fourier series for $f(x) = 1 - x^2$ in (-1, 1) (5)

C) Find a, b, c, d, e if,

$f(z) = (ax^4+bx^2y^2+cy^4+dx^2-2y^2)+i(4x^3y-exy^3+4xy)$ is analytic (5)

D) Prove that $\nabla\left(\frac{1}{r}\right) = -\frac{r}{r^3}$ (5)

Q.2) A) If $f(z) = u + iv$ is analytic and $u + v = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$, find $f(z)$ (6)

B) Find inverse Z-transform of $f(z) = \frac{z+2}{z^2-2z+1}$ for $|z| > 1$ (6)

C) Find Fourier series for $f(x) = \sqrt{1-\cos x}$ in $(0, 2\pi)$

Hence, deduce that $\frac{1}{2} = \sum_1^{\infty} \frac{1}{4n^2-1}$ (8)

Q.3) A) Find $L^{-1}\left\{\frac{1}{(s-2)^4(s+3)}\right\}$ using Convolution theorem (6)

B) Prove that $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = (3x^2-1)/2$ are orthogonal over (-1, 1) (6)

C) Verify Green's theorem for $\int_c \overline{F}.\overline{dr}$ where $\overline{F} = (x^2 - y^2)i + (x+y)j$ and c is the triangle with vertices (0,0), (1,1), (2,1) (8)

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Q.4) A) Find Laplace Transform of $f(t) = |sinpt|, t \geq 0$ (6)

B) Show that $\bar{F}$ = (ysinz-sinx) i +(xsinz+2yz) j + (xycosz+y^2) k is irrotational.

Hence, find its scalar potential. (6)

C) Obtain Fourier expansion of $f(x) = x + \frac{\pi}{2}$ where $-\pi < x < 0$

$= \frac{\pi}{2} - x$ where $0 < x < \pi$

Hence, deduce that (i) $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots$

(ii) $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots$ (8)

Q.5) A) Using Gauss Divergence theorem to evaluate $\iint_s \bar{N}.\bar{F}ds$ where $\bar{F} = 4xi - 2y^2j + z^2k$

and S is the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$ (6)

B) Find $Z\{2^k cos(3k+2)\}, k \geq 0$ (6)

C) Solve $(D^2+2D+5)y = e^{-t}sint$, with $y(0) = 0$ and $y'(0) = 1$ (8)

Q.6) A) Find $L^{-1}\left\{\tan^{-1}\left(\frac{2}{s^2}\right)\right\}$ (6)

B) Find the bilinear transformation which maps the points 2, i, -2 onto points 1, i, -1 by using cross-ratio property. (6)

C) Find Fourier Sine integral representation for $f(x) = \frac{e^{-ax}}{x}$ (8)

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