(Time: 3 hours) **Max Marks: 80**

Note: (1) Question No. 1 is Compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Figures to the right indicate full marks.

Q1.

a) Solve $(x^2+1)dx + (2+y^2)\,dy = 0$ 5

b) Solve $\frac{d^2y}{dx^2} - y = e^{2x} + \sin 2x$ 5

c) Using Euler's method find the approximate value of y at $x = 0.5$ taking $h = 0.1$, $\frac{dy}{dx} = 2 + \sqrt{xy}$, $y(0) = 1$ 5

d) Change the order of integration $I = \int_0^1 \int_x^1 f(x,y)\; dy\, dx$ 5

Q2.

a) Solve $(D^2 - 2D + 1)y = 3x^2$ 6

b) Solve $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$ 6

c) Evaluate $\int_1^7 \frac{6}{x}\, dx$ by using 8
(i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule

Q3.

a) Using the Rule of DUIS prove that $\int_0^\infty \frac{\log(1+ax^2)}{x^2}\, dx = \pi\sqrt{a}$ where $a \geq 0$ hence deduce $\int_0^\infty \frac{\log(1+x^2)}{x^2}\, dx = \pi$ 6

b) Evaluate $\int\int x\; dxdy$ over the region bounded by X-axis, line $x = 4$ and the parabola $x^2 = 4y$ 6

c) Evaluate $\int_0^a \int_0^{b\left(1-\frac{x}{a}\right)} \int_0^{c\left(1-\frac{x}{a}-\frac{y}{b}\right)} z\, dz\, dy\, dx$ 8

Q4.

a) Find area of one lemniscate $r^2 = 4\cos 2\theta$ 6

b) Using Runge-Kutta method of fourth order find the approximate value of y at $x = 0.1$ taking $h = 0.1$, $\frac{dy}{dx} = x + \sqrt{y}$ & y(0)=1 6

c) Evaluate $\int_0^1 \sqrt{9-9x}\; dx \cdot \int_0^\infty \frac{2}{(1+x^2)^{5/2}}\; dx$ 8

Q5.

a) Evaluate $\int\int\int (x^2+y^2+z^2)dx\, dy\, dz$ over the first octant of the sphere $x^2 + y^2 + z^2 = 1$ 6

b) Solve $(x^2y^2+2)ydx + (2-2x^2y^2)xdy = 0$ 6

c) Solve by method of Variation of parameter $\{(D+1)(D-1)\}y = e^x \sin x$ 8

Q6.

a) Using Euler's modified method find the approximate value of y at $x = 1.2$ taking $h = 0.2$, $\frac{dy}{dx} = \log_e(x+y)$, $y(1) = 2$ correct upto 4 decimal places 6

b) Find the length of cardioide $r = (1 - \cos\theta)$. 6

c) Change to polar coordinates and evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$ 8

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