14/05/2025 FE(ALL BRANCHES) SEM-II C SCHEME EM-II QP CODE: 10081982

(Time: 3 hours) **Max Marks: 80**

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

a) Solve $(2x^2+3y^2-7)x\,dx+(3x^2+2y^2-8)y\,dy=0$ 5

b) Solve $\frac{d^2y}{dx^2}-y=e^{2x}+\sin 2x$ 5

c) Using Euler's method find the approximate value of y at $x=0.5$ taking $h=0.1$, $\frac{dy}{dx}=2+\sqrt{xy}$, $y(0)=1$ 5

d) Change the order of integration $I=\int_0^2\int_{\sqrt{2x}}^2 f(x,y)\;dy\,dx$ 5

Q2.

a) Solve $(D^3-2D+4)y=3x^2-5x$ 6

b) Solve $\cos x\frac{dy}{dx}+y\sin x=\sec^2 x$ 6

c) Evaluate $\int_0^6 e^x\;dx$ by using 8

(i) Trapezoidal rule, (ii) Simpson's 1/3rd rule, (iii) Simpson's 3/8th rule

Q3.

a) Using the Rule of DUIS prove that $\int_0^\infty \frac{\log(1+ax^2)}{x^2}\,dx=\pi\sqrt{a}$ 6

where $a\geq 0$ hence deduce $\int_0^\infty \frac{\log(1+x^2)}{x^2}\,dx=\pi$

b) Evaluate $\int\int xy\;dxdy$ over the region bounded by X-axis, line $x=2a$ and the parabola $x^2=4y$ 6

c) Evaluate $\int_0^a\int_0^{b\left(1-\frac{x}{a}\right)}\int_0^{c\left(1-\frac{x}{a}-\frac{y}{b}\right)} z\,dz\,dy\,dx$ 8

Q4.

a) Find area of one loop of lemniscate $r^2 = a^2 \cos 2\theta$ 6

b) Using Runge-Kutta method of fourth order find the approximate value of y at $x = 0.1$ taking $h = 0.1$, $\frac{dy}{dx} = x + \sqrt{y}$ & y(0)=1 6

c) Evaluate $\int_0^3 \sqrt{3x - x^2}\ dx \cdot \int_0^\infty \frac{1}{(1+x^2)^{3/2}}\ dx$ 8

Q5.

a) Evaluate $\int\int\int (x^2 + y^2 + z^2) dx\, dy\, dz$ over the first octant of the sphere $x^2 + y^2 + z^2 = a^2$ 6

b) Solve $(x^2y^2 + 2)ydx + (2 - 2x^2y^2)xdy = 0$ 6

c) Solve by method of Variation of parameter 8

$(D^2 - 2D + 1)y = e^x \sin x$

Q6.

a) Using Euler's modified method find the approximate value of y at $x = 1.2$ taking $h = 0.2$, $\frac{dy}{dx} = \log_e(x + y)$, $y(1) = 2$ correct upto 4 decimal places 6

b) Find the length of cardioide $r = a(1 - \cos\theta)$ which lies outside the circle $r = a\cos\theta$. 6

c) Change to polar coordinates and evaluate $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$ 8

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