(Time: 3 hours) **Max Marks: 80**

Note: (1) Question No. 1 is Compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Figures to the right indicate full marks.

Q1.

a) Solve $(2x^2 + 3y^2 - 7)x\,dx + (3x^2 + 2y^2 - 8)y\,dy = 0$ **5**

b) Solve $(D^2 - 4D + 4)y = e^{2x} + cos3x$, where $D \equiv \frac{d}{dx}$ **5**

c) Evaluate $\int_0^\infty x^3\, e^{-4x^2} dx$ **5**

d) Change the order of integration $\mathrm{I} = \int_0^a \int_{-a+\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} f(x, y)\, dx\; dy$ **5**

Q2.

a) Evaluate $\mathrm{I} = \int\int\int \frac{z^2 dxdydz}{x^2 + y^2 + z^2}$ over the volume of the sphere **6**

$x^2 + y^2 + z^2 = 2$

b) Find the length of the cardioid $r = a(1 + \cos\theta)$ which lies outside the circle $r + a\cos\theta = 0$ **6**

c) Solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ by using the method of Variation of parameters. **8**

Q3.

a) Prove that $\int_0^\infty \frac{1-\cos ax}{x} e^{-x}\, dx = \frac{1}{2}\log(1 + a^2)$, assuming the validity of differentiation under the integral sign. **6**

b) Evaluate $\mathrm{I} = \int\int y\, dx\, dy$ over the area bounded by **6**

$x = 0, y = x^2, x + y = 2$ in the first quadrant.

c) Evaluate the integral **8**

$\mathrm{I} = \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}}\, dx\, dy\, dz.$

Q4.

a) Solve $\cos x \frac{dy}{dx} + y \sin x = \sec^2 x$ **6**

b) Solve $(D^2 + 3D + 2)y = e^{e^x}$ **6**

c) Prove that $\int_0^1 \frac{x^2 dx}{\sqrt{(1-x^4)}} \cdot \int_0^1 \frac{dx}{\sqrt{(1+x^4)}} = \frac{\pi}{4\sqrt{2}}$. **8**

Q5.

a) Change the integral to polar coordinate and evaluate **6**

$$I = \int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)dydx$$

b) Find area of one loop of the lemniscate $r^2 = a^2 \cos 2\theta$ **6**

c) Solve $\frac{dx}{dy} - xy = x^3y^3$ **8**

Q6.

a) Solve $(D^2 - 4)y = x \sinh x$ **6**

b) Solve $(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0$ **6**

c) Change the order of integration and evaluate **8**

$$I = \int_0^a \int_y^{\sqrt{ay}} \frac{x}{x^2+y^2} dxdy$$

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