03/06/2025 FE (SEM-I) ALL BRANCHES C-SCHEME EM-I QP CODE: 10084980

**(Time: 3 hours)** **Max.Marks:80**

NB: (1) Question No.1 is compulsory
(2) Answer any three questions from Q.2 to Q.6
(3) Figures to the right indicate full marks.

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|---|---|---|---|
| 1 | a) | Prove that $\tanh^{-1}(\sin\theta) = \cosh^{-1}(\sec\theta)$ | 5 |
| | b) | Prove that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary. | 5 |
| | c) | Find the nth derivative of cos5x. cos3x.cosx. | 5 |
| | d) | Prove that log secx $= \frac{x^2}{2} + \frac{x^4}{12} + \ldots$ | 5 |
| 2 | a) | Find all values of $(1+i)^{\frac{1}{3}}$ and show that the continued product is $1+i$ | 6 |
| | b) | Find non-singular matrices P&Q such that PAQ is in normal form where $A = \begin{bmatrix} 2 & -2 & 3 \\ 3 & -1 & 2 \\ 1 & 2 & -1 \end{bmatrix}$ | 6 |
| | c) | Find the maximum & minimum values of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. | 8 |
| 3 | a) | If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{zx}\right)$ then prove that $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} + z^2\frac{\partial u}{\partial z} = 0$. | 6 |
| | b) | Test the consistency and solve if consistent x + y + z = 3, x + 2y + 3z = 4, x + 4y + 9z = 6. | 6 |
| | c) | If $y = e^{tan^{-1}x}$ then prove that $(1+x^2)y_{n+2} + ((2n+2)-1)xy_{n+1} + (n^2+n)y_n = 0$ | 8 |
| 4 | a) | If $z = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$ then prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$ | 6 |
| | b) | Investigate for what values of $\lambda$ & $\mu$ the equations $2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite solutions. | 6 |
| | c) | Prove that $logtan\left(\frac{\pi}{4} + \frac{ix}{2}\right) = \frac{1}{2}log(1) + i\,tan^{-1}(sinh\,x)$ | 8 |

5 a) Find tanhx if 5 sinhx – coshx = 5 6

b) If $u = \sin^{-1}\left(\frac{x+y}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}\right)$ then prove that 6

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\sin u}{4}\left[\frac{1-2\cos^2 u}{\cos^3 u}\right]$$

c) Using Newton Raphson method, find approximate root of $x^3 - 2x - 5 = 0$ (correct up to three places of decimals.) 8

6 a) Prove that $\tan 5\theta = \frac{\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}$ 6

b) If $z = x^2y + y^2, x = logt, y = e^t, find \frac{dz}{dt}\ at\ t = 1$ 6

c) Solve the following systems of equations by Gauss-seidel method 8

$20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$

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