Paper / Subject Code: N10411 / Applied Mathematics-I

Question 1 is compulsory.
Attempt any 3 from questions 2 to 6.
A Scientific Calculator is allowed to use.

Applied Mathematics-I
Revised Course (NEP-2020)
(June 2025)

Marks-60
Duration-02 Hours

03/06/2025 FE (SEM-I) ALL BRANCHES (NEP-2020) AM-I QP CODE: 10091911

1. Attempt any **Five** questions. **(Compulsory Problem)**

(a) Prove that $\sqrt{1+\operatorname{cosec}(\theta/2)} = (1-e^{i\theta})^{-1/2} + (1-e^{-i\theta})^{-1/2}$. 3

(b) Find n^{th} derivative of $\frac{2}{(x-1)(x-2)(x-3)}$. 3

(c) If $5\sinh x - \cosh x = 5$, find $\tanh x$. 3

(d) Show that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary and hence find A^{-1}. 3

(e) If $u = 2(ax+by)^2 - k(x^2+y^2)$ and $a^2+b^2 = k$, find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. 3

(f) Using Newton-Raphson Method, find the real root of $x^3 - 2x - 5 = 0$ correct to three decimal places. 3

2. (a) If $\tan[\log(x+iy)] = a+ib$, prove that **4**

$$\tan[\log(x^2+y^2)] = \frac{2a}{1-a^2-b^2} \quad \text{when } a^2+b^2 \neq 1.$$

(b) Examine the function $f(x,y) = y^2 + 4xy + 3x^2 + x^3$ for extreme values. 5

(c) If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. 6

3. (a) Reduce the following matrix to normal form and find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$. **4**

(b) Solve $x^6 + 1 = 0$. 5

(c) Apply Gauss-Seidel iteration method to solve the following linear equations up to the three iterations. 6

$$20x + y - 2z = 17,\ 3x + 20y - z = -18,\ 2x - 3y + 20z = 25.$$

4. (a) Expand $\sin^5\theta\cos^3\theta$ in a series of sines of multiples of θ. **4**

(b) Investigate for what values of λ and μ the equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ 5

Will have (i) No Solution (ii) Unique Solution (iii) Infinitely Many Solutions.

(c) If $y = a\cos(\log x) + b\sin(\log x)$, prove that 6

$$x^2 y_{n+2} + (2n+1)x y_{n+1} + (n^2+1) y_n = 0$$

5. (a) If $\cos\alpha + \cos\beta + \cos\gamma = 0$, $\sin\alpha + \sin\beta + \sin\gamma = 0$, prove that **4**

i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$

ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$

(b) If $\alpha + i\beta = \tanh\left(x + \frac{i\pi}{4}\right)$, prove that $\alpha^2 + \beta^2 = 1$. 5

(c) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$. 6

6. (a) Find all the roots of $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{3/4}$, hence find the continued product of the roots. **4**

(b) Show that the minimum value of $u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$ is $3a^2$. 5

(c) Find non-singular matrices P & Q such that PAQ is in normal form & hence find rank of the matrix A and obtain A^{-1}. 6

$$A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$
