

University of Mumbai
Examinations Summer 2022

Time: 2hour 30 minutes Max. Marks: 80

Q1.	Choose the correct option for following questions. All the Questions are compulsory and carry equal marks																
1.	Evaluate $\int_0^{1+i} (x^2 + iy) dz$ along the path $y = x$																
Option A:	$\frac{-1+5i}{6}$																
Option B:	6																
Option C:	-1																
Option D:	0																
2.	The function $f(z) = \frac{z^2}{(z+2)(z-1)^2}$ has																
Option A:	simple pole at $z = -2$ & pole of order 2 at $z = 1$																
Option B:	simple pole at $z = 2$																
Option C:	simple pole at $z = 0$ & pole of order 1 at $z = 2$																
Option D:	simple pole at $z = 0$																
3.	If the line of regression of Y on X is $(Y - 33) = (0.72)(X - 33)$ then the approximate value of Y for $X = 38$ is																
Option A:	20																
Option B:	37																
Option C:	1																
Option D:	10																
4.	If for the non-repeated ranks $\sum D^2 = 8$ & $n = 5$ then the rank correlation coefficient (R) is																
Option A:	0																
Option B:	0.6																
Option C:	7																
Option D:	1																
5.	Find k if pdf of rv X is																
	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr> <tr> <td>P(X=x)</td><td>k</td><td>3k</td><td>5k</td><td>7k</td><td>9k</td><td>11k</td><td>13k</td></tr> </table>	X	0	1	2	3	4	5	6	P(X=x)	k	3k	5k	7k	9k	11k	13k
X	0	1	2	3	4	5	6										
P(X=x)	k	3k	5k	7k	9k	11k	13k										
Option A:	1/5																
Option B:	2																
Option C:	1/49																
Option D:	7																
6.	Find E(X) if $f(x) = x$ $0 \leq x \leq 1$																
Option A:	2																
Option B:	1																
Option C:	1/3																
Option D:	0																

7.	If $u = (-2, 3, 4)$ and $v = (3, -2, 3)$ then their dot product is
Option A:	3
Option B:	1
Option C:	2
Option D:	0
8.	If $u = (2, 1, 0)$ then $\ u\ $
Option A:	$\sqrt{50}$
Option B:	$\sqrt{3}$
Option C:	$\sqrt{5}$
Option D:	1
9.	The matrix form of the Quadratic form $x^2 - 2y^2 + 3z^2 - 4xy + xz - 2yz$ is
Option A:	$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 1 & -2 & 1/2 \\ -2 & -2 & -1 \\ 1/2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
Option B:	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & -2 & -1 \\ 1/2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix}$
Option C:	$\begin{bmatrix} 1 & -2 & 1/2 \\ -2 & -2 & -1 \\ 1/2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix}$
Option D:	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x & y & z \end{bmatrix}$
10.	Find the extremals of $\int_{x_1}^{x_2} \frac{y'^2}{x^2} dx$
Option A:	$y = c_1 x^5 + c_2$
Option B:	$y = c_1 x + c_2$
Option C:	$y = c_1 x^3 + c_2$
Option D:	$y = c_1 x^6 + c_2$

Q2	Solve any Four out of Six	5 marks each
A	Using residue theorem evaluate $\int_C \frac{3z^2 + z}{z^2 - 1} dz$ where C is the circle $ z = 2$	
B	Fit a straight line to the following data X: 5 6 7 8 9 Y: 2 4 5 6 8	
C	For a normal variate with mean 2.5 and standard deviation 3.5. Find the probability that i) $2 \leq x \leq 4.5$ ii) $-1.5 \leq x \leq 5.5$	
D	Find a unit vector in \mathbb{R}^3 orthogonal to both $u = (1, 0, 1)$ and $v = (0, 1, 1)$.	

E	Reduce the quadratic form $6x^2+3y^2+14z^2+4xy+4yz+18xz$ to diagonal form using congruent transformations.
F	Find the extremals of $\int_0^1 (xy + y^2 - 2y^2 y') dx$

Q3	Solve any Four out of Six	5 marks each
A	Evaluate $\int_C \frac{dz}{z^3(z+4)}$ where C is the circle $ z = 2$	
B	Given $6y = 5x + 90$; $15x = 8y + 130$. Find i) \bar{x} and \bar{y} ii) r	
C	Three factories A, B, C produce 30%, 50% and 20 % of the total production of an item. Out of their production 80%, 50% and 10% are defective. An item is chosen at random and found to be defective. Find the probability that it was produced by the factory A.	
D	Construct an orthonormal basis of R^2 Using Gram-Schmidt process to $S = \{(3, 1), (2, 3)\}$	
E	Reduce the quadratic form $x^2+2y^2+2z^2-2xy-2yz+xz$ to canonical form. Also find its rank & signature.	
F	Find the extremals of $\int_{x_0}^{x_1} (2xy - y'^2) dx$	

Q4	Solve any Four out of Six	5 marks each
A	Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the regions i) $ z < 1$ ii) $1 < z-1 < 2$	
B	Calculate R and r from the following data $X : 12 \quad 17 \quad 22 \quad 27 \quad 32$ $Y : 113 \quad 119 \quad 117 \quad 115 \quad 121$	
C	Fit a poission distribution to the following data No. of deaths : 0 1 2 3 4 Frequencies : 123 59 14 3 1	
D	Let V be a set of positive real numbers with addition and scalar multiplication defined as $x + y = xy$ and $kx = x^k$ Show that V is a vector space under this addition and scalar multiplication.	
E	Find the singular value decomposition of $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$	
F	Using Rayleigh-Ritz method , solve the boundary value problem $\int_0^1 (2xy + y^2 - y'^2) dx ; 0 \leq x \leq 1$ given $y(0) = y(1) = 0$	