

19 NOV 2025 TE EXTC (SEM-V) C SCHEME RSA QP CODE: 10095158

Time: 3 Hours

Max Marks: 80

**N.B: 1. Question No 1 is Compulsory****2. Answer any 3 questions from the remaining questions**

- Q1. a. State and prove Chebyshev's inequality. 05  
 b. Define covariance and correlation. What are the conditions for two random variables to be uncorrelated? 05  
 c. A random variable X has the probability density function 05  

$$f(x) = \frac{x}{8}, \text{ for } 0 \leq x \leq 4. \text{ Find the mean and variance of X.}$$
  
 d. State and prove Bayes' Theorem 05  
 e. Define Rayleigh distribution and write the expressions for its mean and variance. 05

- Q2. a. Define central limit theorem. What is the significance of central limit theorem? 10  
 b. The joint probability distribution function of (X, Y) is 10

$$f_{XY}(x, y) = \begin{cases} \frac{2}{3}(x + 2y); & 0 \leq x \leq 1; 0 \leq y \leq 1 \\ 0; & \text{elsewhere} \end{cases}$$

Find the conditional density function  $f_X(x/y)$ , Check whether X and Y are independent.

- Q3. a. In a binary communication a 0 or 1 is transmitted. Because of channel noise, a 0 can be received as a 1 and vice versa. Let T0 and T1 denote the event of transmitting 0 and 1, respectively. Let R0 and R1 denote the events of receiving 0 and 1, respectively. Let  $P(T0) = 0.7$ ,  $P(R1/T0) = 0.4$  and  $P(R0/T1) = 0.3$ . 10  
 i. Find the probability that '0' is received.  
 ii. If a 0 was received, what is the probability that a 0 was sent?  
 iii. Calculate the probability of error  
 iv. Calculate the probability of transmitted signal is correctly read at the receiver  
 b. Define autocorrelation function and state its properties 10

- Q4. a. Prove that for a linear time invariant system, if the input is a WSS process, then output is also WSS Process. 10  
 b. A company manufactures an electronic device that operates over a wide temperature range. It is known that higher temperatures reduce the lifetime of the device. To study this effect, the following data were obtained. Find the equation of the regression line from the following data and Predict the output when temperature is 65°C. 10

Temperature(°C)	10	20	30	40	50	60	70	80	90
Lifetime (hours)	42	36.5	28.5	22	17.6	11.7	6.9	3.4	0.5

- Q5. a. If the WSS process  $X(t)$  is given by  $X(t) = 10 \cos(\omega t + \theta)$ , where  $\theta$  is uniformly distributed over  $(-\pi, \pi)$ . Prove that  $X(t)$  is correlation ergodic. 10
- b. Derive the moment generating function for Poisson distribution. By using the moment generating function, derive the mean and variance of Poisson distribution. 10

- Q6. a. The joint pdf of  $(X, Y)$  is given by 10

$$f_{XY}(x, y) = \begin{cases} k e^{-(x+2y)}; & x \geq 0 \text{ and } y \geq 0 \\ 0; & \text{elsewhere} \end{cases}$$

Find  $k$ ,  $E(XY)$ , Are  $X$  &  $Y$  independent, uncorrelated and orthogonal

- b. If mean  $E[X] = 6/25$ , find  $a$ ,  $b$ , mean and variance of  $Y$  if  $Y = X^2 + 2$  10

X	-2	-1	0	1	2	3
P(X)	0.2	3a	0.1	6b	0.1	0.24

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