Time: 3 Hours **Total Marks: 80**

N.B. : (1) Question No. 1 is compulsory.
(2) Attempt any three questions out of the remaining five questions

Q.1 (a) Given the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 2 & 5 & 7 \end{bmatrix}$ find null space of A and rank of A . **(5)**

(b) A box contains 6 red balls and 4 blue balls. Three balls are drawn **with replacement**. Let **X = number of red balls drawn**. **(5)**
Find:
(i) Probability Distribution of X
(ii) Cumulative Distribution function of X

(c) The weights (in kg) of 15 students are: **(5)**
46, 48, 49, 51, 52, 52, 53, 54, 55, 56, 58, 60, 61, 95, 102
Identify the outlier if the outlier lies more than ±2 standard deviations away from the mean.

(d) Obtain the Hessian Matrix for the function **(5)**

$$Z = 18x_1x_2 + 10x_1 - 34x_3 + 70x_1^2 + 40x_2^2 + 50\,x_3^2$$

Q.2 (a) Find Singular Value of Decomposition of matrix $A = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 5 & 0 \end{bmatrix}$ **(10)**

(b) A company claims that the mean lifetime of a battery is 500 hours. **(10)**
A sample of 25 batteries gives:
- Sample mean = 480 hours
- Sample SD = 40 hours

Test at **5% significance** whether the population mean is different from 500 hours. (Given t=2.145 at 5% level of significance for a two-tailed test)

Q.3 (a) Given the following data, plot a "less than" Ogive and use it to estimate the median score. **(10)**

Class Interval	Frequency
00-10	4
10-20	8
20-30	12
30-40	10
40-50	6

(b) Conduct a two tailed F Test on the following samples: **(10)**
Sample 1: Variance = 109.63, sample size = 41.
Sample 2: Variance = 65.99, sample size = 21
(Given $F_{((40,20),0.025)=2.287}$, $F_{((40,20),0.975)=0.4836}$))

Q.4 (a) Apply **Linear Discriminant Analysis (LDA)** for the following data of two classes: **(10)**

Class C1 – Apples (weight, colour intensity):
C1 = {(150, 8), (160, 7), (155, 9), (165, 8)}
Class C2 – Oranges (weight, colour intensity):
C2 = {(120, 5), (130, 4), (125, 6), (135, 5)}
Find:

i. The **projection vector**
ii. The **discriminant function**

(b) You are given the following data set of two features (variables) for 5 students: **(10)**

Student	A	B	C	D	E
Skill Score (X1)	90	80	70	60	50
Productivity Score (X2)	85	70	65	60	55

Compute:

i. Covariance between X1 and X2
ii. Correlation coefficient
iii. Covariance matrix

Q.5 (a) Minimize the function $f(x_1, x_2) = x_1x_2 - 8x_1 - 6x_2 + 6x_2 + x_1^2 + x_2^2$ subject to $2x_1 + x_2 = 12, \quad x_1, x_2 \geq 0$ **(10)**

(b) Find the minimizer of $f(x) = (x-3)^2 + 2$ using bisection method in (0,6) with an accuracy of 0.1 **(10)**

Q.6 Attempt any four **(20)**

(a) Plot the graphs of the following functions over the interval x∈[−3,3] **(5)**
(a) e^{-x} (b) e^{-x^2}

(b) Explain the curse of dimensionality with reference to: **(5)**

i. Distance metrics
ii. Volume scaling
iii. Overfitting in machine learning

(c) Write short notes on **(5)**

1. **Principal Component Regression (PCR)**
2. **Linear Discriminant Analysis**

(d) Find column Space of $A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 6 & 9 \end{bmatrix}$ **(5)**

(e) Explain and differentiate with examples: **(5)**

i. **K-Nearest Neighbors (KNN)**
ii. **Support Vector Classifier (SVC)**

(f) Write short notes on Non gradient based optimization technique. **(5)**

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