Time: 3 hour **Marks: 80**

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

Q1	**Attempt All questions**	**Marks**
A	If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ then find the eigen values of A^2	**5**
B	Find Laplace transform of $f(t) = te^t \sin 2t$	**5**
C	Find the half range Sine Series for $f(x) = x^2$, where $x \in (-\pi, \pi)$	**5**
D	Determine the constant a, b, c, d if $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$ is analytic.	**5**
Q2		
A	Using Green's theorem in a plane to evaluate the line integral $\oint_C (xy + y^2)dx + x^2dy$ Around the boundary of the region defined by $y=x^2$ and $y=x$	**6**
B	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	**6**
C	Show that the function $v = e^x siny$ satisfies Laplace's equation, also find analytic function.	**8**
Q3		
A	Prove that $\bar{F} = (x^2 + xy^2)i + (y^2 + x^2y)j$ is irrotational.	**6**
B	Prove that the function $(x^3 - 3xy^2 + 3x) + i(3x^2y - y^3 + 3y)$ is analytic	**6**
C	Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$	**8**
Q4		
A	Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ Where $\bar{F} = x^2i + xyj$ where C is the boundary of the rectangle x=0 , y=0, x=a, y=b	**6**
B	Evaluate $\int_0^\infty \frac{e^{-2t} cos2t}{t} dt$, using Laplace transforms	**6**
C	Using Convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+9)(s^2+4)}\right]$	**8**

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Q5

A Find $L\{e^{-t}sin2t\ cos3t\}$ **6**

B Prove that the vector field $\bar{F}$ on R^3 defined by **6**

$$\bar{F} = (y^2cosx + z^3)\hat{\imath} + (2ysinx - 4)\hat{\jmath} + (3xz^2 + 2)\hat{k}$$

is conservative and find its scalar potential.

C Find the Fourier Series for $f(x) = \frac{3x^2-6x\pi+2\pi^2}{12}$ in $0 \leq x \leq 2\pi$ **8**

Q6

A Obtain Fourier series expansion of $f(x) = 9 - x^2$ in (-3, 3) **6**

B Prove that the matrix A is diagonalisable, find the transforming matrix and the diagonal matrix. **6**

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

C

i) Find $L^{-1}\left\{\log\left(\frac{s+1}{s^2+2}\right)\right\}$ **4**

ii) Find $L^{-1}\left\{\frac{1}{s^2+4s+5}\right\}$ **4**

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