

10 NOV 2025 SE SEM-III C SCHEME EXTC EM-III QP CODE: 10093307

Time: 3 hour

Max. Marks: 80

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

Q1 Attempt All questions

A If  $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & -2 \end{bmatrix}$  then find the eigen values of  $A^2$  5

B Find Laplace transform of  $f(t) = te^t \cos 2t$  5

C Find the half range Sine Series for  $f(x) = x$ , where  $x \in (-\pi, \pi)$  5

D Determine the constant a, b, c, d if  $f(z) = x^2 + 2axy + by^2 + i(cx^2 + 2dxy + y^2)$  is analytic. 5

Q2

A Using Green's theorem in a plane to evaluate the line integral 6

$$\oint_C (xy + y^2)dx + x^2dy$$

Around the boundary of the region defined by  $y=x^2$  and  $y=x$

B Find the Eigen values and Eigen vectors of the matrix 6

$$A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

C Show that the function  $v = e^x \sin y$  satisfies Laplace's equation, also find analytic function. 8

Q3

A Prove that  $\vec{F} = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$  is irrotational. 6

B Find the analytic function whose real part is  $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$  6

C Verify Cayley-Hamilton theorem for the matrix A and hence find  $A^{-1}$  and  $A^4$  8

where  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

**Q4**

**A** Using Stokes theorem to evaluate  $\int_C \vec{F} \cdot d\vec{r}$  **6**  
 Where  $\vec{F} = x^2\hat{i} + xy\hat{j}$  where C is the boundary of the rectangle  
 $x=0, y=0, x=a, y=b$

**B** Evaluate  $\int_0^\infty \frac{e^{-t} \sin 2t}{t} dt$ , using Laplace transforms **6**

**C** Using Convolution theorem find  $L^{-1} \left[ \frac{s^2}{(s^2+1)(s^2+4)} \right]$  **8**

**Q5**

**A** Find  $L \{ e^{-4t} \sin 3t \cos 2t \}$  **6**

**B** Prove that the vector field  $\vec{F}$  on  $R^3$  defined by **6**  
 $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + (3xz^2 + 2)\hat{k}$   
 is conservative and find its scalar potential.

**C** Find the Fourier Series for  $f(x) = \left(\frac{\pi-x}{2}\right)^2$  in  $0 \leq x \leq 2\pi$  **8**

**Q6**

**A** Obtain Fourier series expansion of  $f(x) = 4 - x^2$  in  $(-2, 2)$  **6**

**B** Prove that the matrix A is diagonalisable, find the transforming matrix and the diagonal matrix. **6**

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

**C** **4**  
 i) Find  $L^{-1} \left\{ \log \left( \frac{s^2+1}{s^2+2} \right) \right\}$

ii) Find  $L^{-1} \left\{ \frac{1}{s^2+2s+10} \right\}$  **4**