Time: 3 hour **Max. Marks: 80**

Note: 1) Question 1 is compulsory.

2) Attempt any 3 questions from Question 2 to Question 6

3) Figures to the right indicate full marks.

		Marks
Q1	Attempt All questions	
A	If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$ then find the eigen values of $A^{-1} + A^2$	5
B	Find Laplace transform of $f(t) = t\{\sqrt{1 + sint}\}$	5
C	Find the Fourier Series for $f(x) = x^2$, where $x \in (-\pi, \pi)$	5
D	Prove that $f(z) = logz$ is analytic, also find its derivative.	5
Q2		
A	Using Green's theorem in a plane to evaluate $\oint_C (x^2 - y^2)dx + (x + y)dy$ and C is the triangle with vertices (0, 0) , (1, 1) and (2, 1)	6
B	Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$	6
C	Show that the function u = sinx coshy + 2cosx sinhy + $x^2 - y^2$ + 4xy satisfies Laplace's equation, also find analytic function.	8
Q3		
A	If $\bar{F} = (y^2 - z^2 + 3yz - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ show that $\overline{F}$ is irrotational and solenoidal.	6
B	If v = $e^x siny$, prove that v is a harmonic function. Also find the corresponding harmonic conjugate.	6
C	Prove that the matrix A is diagonalisable, also find diagonal form and transforming matrix. $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$	8

Q4

A Using Stokes theorem to evaluate $\int_C \bar{F} \cdot d\bar{r}$ — 6

Where $\bar{F} = x^2 i - xyj$ and C is the square in the plane z = 0 and bounded by x = 0 , y = 0, x = a and y = a

B Evaluate $\int_0^{\infty} \frac{\cos at - \cos bt}{t} dt$, using Laplace transforms — 6

C Using Convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$ — 8

Q5

A Find $L\left\{\int_0^t u \sin 4u \, du\right\}$ — 6

B Consider the vector field $\bar{F}$ on $\mathbb{R}^3$ defined by — 6

$$\bar{F}(x,y,z) = y\,\hat{\imath} + (z\cos(yz) + x)\,\hat{\jmath} + (y\cos(yz))\,\hat{k}$$

Show that $\bar{F}$ is conservative and find its scalar potential.

C Find the Fourier Series for $f(x)$ in $(-\pi, \pi)$ where — 8

$$f(x) = x + \frac{\pi}{2} \qquad -\pi \le x \le 0$$
$$= \frac{\pi}{2} - x \qquad 0 \le x \le \pi$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots$

Q6

A Obtain Fourier series expansion of $f(x) = 4 - x^2$ in (-2, 2) — 6

B Verify Cayley-Hamilton theorem for the matrix A and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ — 6

C

i) Find $L^{-1}\left\{\log\left(\sqrt{\frac{s+a}{s+b}}\right)\right\}$ — 4

ii) Find $L^{-1}\left\{\frac{1}{s^2+2s+5}\right\}$ — 4

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