Time: 2 hours **Max Marks: 60**

Note: (1) Question No. 1 is Compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Figures to the right indicate full marks.
(4) Statistical Table to be provided

Q.1. Attempt any **Five Questions** from the following.

a) Find the sum and product of the eigenvalues of $A = \begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix}$ 3

b) Construct the dual of the following LPP 3

Maximise $Z = 5x_1 + 4x_2$

Subject to constraint $2x_1 + 6x_2 \leq 5$

$7x_1 + 3x_2 \leq 2$

$x_1,\ x_2 \geq 0$

c) Find value of a_n in the expansion of Fourier series for $f(x) = x^2\ in\ (-\pi, \pi)$ 3

d) Calculate first 2 raw moments for probability density function 3

$f(x) = \begin{cases} xe^{-x} & x \geq 0 \\ 0 & Otherwise \end{cases}$

e) What is the remainder when 5^{11} is divided by 7 3

f) Obtain Spearman's rank correlation coefficient for following data 3

X	18	17	13	17	19	21
Y	21	22	14	17	18	13

Q.2.

a) The number of traffic accidents that occur on a particular stretch of road during a month follows a Poisson distribution with a mean of 7. Find the probability of observing exactly three accidents on this stretch of road next month. 4

b) Calculate the correlation coefficient for the following heights of fathers(X) and their sons(Y) 5

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

c) Expand Fourier series for $f(x) = x^2\ in\ (0, 2)$ 6

Q.3.

a) Fit a straight line of the form y=a+bx to the following data 4

X	1	3	5	7	8	10
Y	8	4	5	7	8	4

b) By using Chinese Remainder theorem, solve $x \equiv 3(mod\ 4),\ \ x \equiv 5(mod\ 7)$ 5

c) Find eigen values and eigen vectors of $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ 6

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Q4.

a) A random variable X has the following probability distribution 4

X	1	2	3	4
P(X=x)	0.2	0.3	0.3	0.2

Find moment generating function about origin of x, hence find first two raw moments.

b) The distribution of wait times at a drive-through restaurant is approximately normal with mean 185 seconds and standard deviation 15 seconds. find the percentage of wait times that are 5
i. between 155 and 215 seconds.
ii. more than 170 seconds.

c) Check whether the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is diagonalizable, if yes, find the diabolizing matrix P and the diagonal matrix D. 6

Q5.

a) Verify Cayley-Hamilton theorem for matrix $A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ 4

b) Maximise $Z = 7x_1 + 5x_2$ 5
Subject to constraint $-x_1 - 2x_2 \geq -6$
$4x_1 + 3x_2 \leq 12$
$x_1,\ x_2 \geq 0$

c) Perform key generation encryption and decryption for RSA Algorithm with the following parameters primes p=3, q= 11 message, M=5 6

Q6.

a) Expand Fourier series for $f(x) = x\ in\ (-\pi,\ \pi)$ 4

b) Obtain the equation of line of regression of X on Y 5

X	21	23	25	27	28	20
Y	8	12	15	17	18	20

c) Using the method of Lagrange's multipliers, solve the following NLPP 6
Maximise $Z = {x_1}^2 + {x_2}^2 + {x_3}^2 - 10x_1 - 6x_2 - 4x_3$
Subject to constraint $x_1 + x_2 + x_3 = 7$ $x_1,\ x_2,\ x_3 \geq 0$

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