Time: 3 Hours **Marks: 60**

N. B. 1) Q.1 is compulsory
2) Attempt any three out of the remaining
3) Figures to the right indicate full marks
4) Assume suitable data if necessary

Q. 1 a) A digital geometry learning platform groups polygons based on the number of sides for automatic classification and retrieval. Let Abe the set of all polygons. A relation R is defined on A as:

$$R = \{(P_1, P_2): P_1 \text{ and } P_2 \text{ have the same number of sides}\}$$

Based on this classification system:

1. Analyze whether the relation $R$is an **equivalence relation** on A.
2. Verify the properties of **reflexivity, symmetry, and transitivity**.
3. Construct suitable equivalence classes for triangles, quadrilaterals, and pentagons.
4. Interpret how equivalence relations help organize geometric data in digital systems. (05M)

b) A software security system uses logical rules to decide whether access should be granted, denied, or forwarded for further verification.
Consider the following logical expressions:

$$((p \to q) \to r)$$
and
$$(p \wedge \neg q) \vee r$$

Using a series of logical equivalences, prove that the two expressions are logically equivalent. Also interpret how logical equivalence helps simplify decision rules in software systems. (05M)

c) A quality control department stores 10 pairs of socks of different colors in a drawer for packaging verification.
Using the **Pigeonhole Principle**, analyze the minimum number of socks that must be Selected randomly to guarantee that at least one matching pair of the same color is obtained. Justify your answer with suitable reasoning. (05M)

Q. 2 a) A pharmaceutical company conducts a drug trial on **1000 people** to analyze the side effects of a newly developed medicine. The following observations are recorded:

- 122 people develop symptom A
- 88 people develop symptom B
- 112 people develop symptom C
- 27 people develop both symptoms $A$and B
- 29 people develop both symptoms $A$and C
- 32 people develop both symptoms $B$and C
- 10 people develop all three symptoms A, B, and C

Using the **Principle of Inclusion and Exclusion**:

1. Analyze how many people develop **at least one symptom**.
2. Determine the number of people who show **exactly one symptom**.
3. Construct a **Venn diagram** and fill all regions appropriately.

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b) A retail store offers seasonal discounts on clothing items. A customer wants to purchase a shirt priced at Rs.1000. The store first provides a **15% discount**, after which an **8% sales tax** is applied to the discounted price.

Based on this shopping scenario:

1. Develop suitable mathematical functions representing the discount process and the tax calculation process.
2. Construct the **composite function** representing the final cost of the shirt.
3. Determine the final amount to be paid by the customer.
4. Analyze how composite functions help automate billing systems in retail applications. (07M)

Q.3 a) A domestic airline company maintains a directed flight network among four cities:

$$V = \{a, b, c, d\}$$

The direct flight connections are represented by the relation:

$$R = \{(a, b), (b, d), (d, c), (d, a)\}$$

The company wants to analyze all possible direct and indirect travel routes, including circular journeys.

Based on the given flight network:

1. Identify and explain the concept of **reachability / transitive closure** used in the graph.
2. Determine all possible ordered pairs of cities connected through:
   - direct routes,
   - routes with layovers, and
   - circular routes.
3. Analyze how graph connectivity helps optimize airline route planning.
4. Represent the final connectivity relation for the graph. (08M)

b) A cloud access-control system organizes permission levels using divisibility-based hierarchy. The permission levels are:

$$A = \{1,2,3,6,12,24,36,72\}$$

A relation $R$is defined on Aas:

$$(x, y) \in R \text{ if } x \mid y$$

Based on this hierarchy:

1. Write all ordered pairs of relation R.
2. Construct the **Hasse Diagram** of the poset.
3. Analyze whether the given poset forms a **lattice**.
4. Justify your answer using suitable meet and join examples. (07M)

Q.4 a) A cryptographic system uses non-zero residue classes modulo 7 for secure key generation. Consider the set:

$$G = \{1,2,3,4,5,6\}$$

under multiplication modulo 7.

Based on this system:

1. Construct the multiplication table of Gunder multiplication modulo 7.
2. Determine the multiplicative inverses of 2,3,5,and 6.
3. Analyze whether Gforms a cyclic group.
4. If it is cyclic, identify at least one generator and justify your answer.

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b) A data analyst observes that the cumulative computational cost of a repeated process is modeled by the series:

$$\sum_{r=1}^{n} r(r+3)$$

The analyst claims that for every positive integer n,

$$\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5), n \geq 1, n \in \mathbb{N}$$

Using the **principle of mathematical induction**:

1. Verify the result for the base case.
2. Prove the inductive step.
3. Conclude that the formula is valid for all $n \geq 1$.
4. Interpret how induction helps validate formulas in algorithm analysis. (07M)

Q.5 a) A computer networking company proposes two communication network layouts represented by graphs G_1 and G_2. The organization wants to verify whether both network structures provide the same connectivity pattern even though the device labels are different.

The graphs are represented as:

$$V(G_1) = \{A, B, C, X, Y, Z\}$$
$$E(G_1) = \{(A, X), (A, Y), (B, X), (B, Y), (C, Y)\}$$
$$V(G_2) = \{1,2,3,4,5,6\}$$
$$E(G_2) = \{(1,4), (1,5), (2,4), (2,5), (3,5)\}$$

Based on the above network structures:

1. Define **isomorphism of graphs**.
2. Analyze whether graphs G_1 and G_2 are **isomorphic**.
3. If they are isomorphic, determine a valid **one-to-one correspondence** between the vertices.
4. Justify your answer by comparing degree sequence and adjacency preservation. (08M)

b) A financial analytics company models the quarterly growth of a digital investment portfolio using a recurrence relation. Let F_n represent the portfolio value during the n^{th} quarter.

The growth pattern is governed by:

$$F_n = 3F_{n-1} + 10F_{n-2} + 7 \cdot 5^n$$

with initial conditions:

$$F_0 = 4, F_1 = -3$$

Based on this predictive model:

1. Solve the given recurrence relation using an appropriate method.
2. Determine the general expression for F_n.
3. Analyze the effect of the exponential forcing term $7 \cdot 5^n$ on the sequence growth.
4. Interpret how recurrence relations are useful in financial forecasting and computational modeling. (07M)

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Q.6 a) A cryptographic system uses arithmetic operations on residue classes modulo 4 for encoding and verification. Consider the set:

$$Z_4 = \{0,1,2,3\}$$

with addition and multiplication defined modulo 4.
Based on this algebraic structure:

1. Construct the addition and multiplication tables for Z_4.
2. Verify whether Z_4satisfies the required ring properties.
3. Analyze the role of additive identity, additive inverse, multiplicative associativity, and distributive laws.
4. Conclude whether Z_4forms a ring under addition and multiplication modulo 4. (08M)

b) A city transport authority is analyzing a road network connecting six important locations A, B, C, D, E,and F. The network is represented by the following graph:
Graph

Based on the given road network:

1. Determine whether the graph has an **Euler Path**.
2. Determine whether the graph has an **Euler Circuit**.
3. Determine whether the graph has a **Hamiltonian Path**.
4. Determine whether the graph has a **Hamiltonian Circuit**.
5. Justify your answer using degree of vertices and graph traversal conditions. (07M)

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