

SE COMP / SEM-III / C-scheme / FH-23 / 2-1-11-2023

(Time: 3 Hours)

Max. Marks: 80

- N.B. (1) Question No. 1 is compulsory.
 (2) Answer any three questions from Q.2 to Q.6.
 (3) Use of Statistical Tables permitted.
 (4) Figures to the right indicate full marks

Q1.

- (a) Find the Laplace transform of $\frac{\cos 2t \sin t}{e^t}$ [5]
 (b) Find k such that $f(z) = \frac{1}{2} \log(x^2 + y^2) + i \tan^{-1} \frac{kx}{y}$ is analytic [5]
 (c) Calculate the Spearman's rank correlation coefficient R [5]
 $X : 10, 12, 18, 18, 15, 40.$
 $Y : 12, 18, 25, 25, 50, 25.$
 (d) Find the inverse Laplace transform of $\log\left(\frac{s^2 + a^2}{s^2 + b^2}\right)$. [5]

Q2.

- (a) A continuous random variable has probability density function
 $f(x) = k(x - x^2), \quad 0 \leq x \leq 1.$
 $f(x) = 0$ otherwise

Find k , mean and variance. [6]

- (b) Find the Laplace transform of $e^{-3t} \int_0^t u \sin 3u \, du$. [6]
 (c) Obtain the Fourier series to represent $f(x) = x^2$ in $(0, 2\pi)$
 Hence show that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ [8]

Q3.

- (a) If the imaginary part of the analytic function $w = u + iv = f(z)$ is
 $V = x^2 - y^2 + \frac{x}{x^2 + y^2}$, then show that $u = -2xy + \frac{y}{x^2 + y^2}$. [6]

- (b) Find inverse Laplace transform of $\frac{2s^2 - 6s + 5}{(s^3 - 6s^2 + 11s - 6)}$ [6]

- (c) Fit a second-degree parabolic curve and estimate y when $x = 10$

x	: 1, 2, 3, 4, 5, 6, 7, 8, 9,	
y	: 2, 6, 7, 8, 10, 11, 11, 10, 9.	[8]

Q4.

- (a) Obtain the Fourier series to represent $f(x) = x^3$ in $(-\pi, \pi)$. [6]

- (b) Find (i) the equation of the lines of Regression (ii) coefficient of correlation for the following data

X :	65, 66, 67, 67, 68, 69, 70, 72.	
Y :	67, 68, 65, 66, 72, 72, 69, 71.	[6]

- (c) Prove that $\int_0^\infty e^{-\sqrt{2}t} \frac{\sin t \sin ht}{t} dt = \frac{\pi}{8}$. [8]

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Q5.

(a) Find the orthogonal trajectories of the family of curves $x^3y - xy^3 = c$. [6]

(b) Find the moment generating function of the distribution

X	:	-2	3	1
P (X=x)	:	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

hence find first four central moments . [6]

(c) Obtain the half range cosine series of $f(x) = x$ in $(0, 2)$

Hence show that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$ [8]

Q6.(a) Using convolution theorem Find the inverse Laplace transform of $\left[\frac{s^2}{(s^2+2)^2} \right]$ [6]

(b) The probability density function of a random variable X is

X	:	1	2	3	4	5	6	7
P (X=x)	:	k	2k	3k	k^2	$k^2 + k$	$2k^2$	$4k^2$

Find k , $p(X < 5)$, $P(X > 5)$ [6]

(c) If $v = 3x^2y + 6xy - y^3$, show that v is harmonic function

And find the corresponding analytic function . [8]