

(3 Hours)

QP. Code: - 10096312

- Note: 1) Question No.1 is compulsory  
2) Attempt any Three from the remaining

Q.1.

- A) Find  $L \left\{ \int_0^t u^n e^{6u} du \right\}$  5
- B) Prove that  $f(z) = e^x(\cos y + i \sin y)$  is analytic everywhere. Hence find  $f'(z)$  5
- C) Find half range sine series of  $f(x) = x^2$  in  $(0, \pi)$  5
- D) If  $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$  then Find an eigen value of 5
- i) A
- ii)  $\text{Adj}(A)$
- iii)  $A^2 - 2A + I$

Q.2.

- A) If  $L[f(t)] = \frac{1}{s^2+1}$  then Find  $L[te^t f(t)]$  6
- B) Find Fourier series for  $f(x) = x^3$ , if  $0 < x < 2\pi$  and  $f(x + 2\pi) = f(x)$  6
- C) Find analytic function  $f(z)$  in terms of  $z$  where  $u = x^2 + y^2 + 5x + y + 2$  8

Q.3.

- A) A string is stretched and fastened to two points distance  $l$  apart. Motion is started by displacing the string in the form  $y = a \sin(\pi x / l)$  from which it is released at time  $t=0$ . Show that the displacement of a point at a distance  $x$  from one end at time  $t$  is given by  $y = a \sin(\pi x / l) \cos(\pi c t / l)$  6
- B) Prove that  $u = y^3 - 3x^2 y$  is harmonic function hence find its harmonic conjugate function 6
- C) Find the Fourier Series for  $f(x) = |x|$  in  $(-\pi, \pi)$  where 8

Q.4.

- A) Evaluate  $\int_0^\infty \left[ \frac{\sin at}{t} \right] dt$  6
- B) Find Inverse Laplace transform of  $\frac{s}{(s-1)(s-2)(s-3)}$  6
- C) Is the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$  Diagonalizable? If so find the Diagonal form of  $A$  and transforming matrix of  $A$  8

Q.5.

- A) If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$  then verify Cayley Hamilton theorem and hence 6

Compute:  $A^5 + A^4 - 18A^3 - 40A^2$

- B) Solve by Crank-Nicholson simplified formula  $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0$ , 6

$0 \leq x \leq 1$  subject to the condition  $u(0, t) = 0, u(1, t) = 100t$ ,

$u(x, 0) = 0$   $h = 0.25$  for one-time step

- C) Find inverse Laplace transform of (i)  $\log \left[ \frac{s+1}{s+5} \right]$  (ii)  $\frac{s-4}{(s+6)^2}$  8

Q.6.

- A) Find the Laplace Transform of  $e^{-t} \int_0^t \sin(u) \cos(u) du$  6

- B) Find the solution of 6

$$4 \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0, 0 < x < 8, \quad u(x, 0) = 4x - \frac{1}{2}x^2, u(0, t) = 0, u(8, t) = 0$$

Taking  $h = 1, k = \frac{1}{8}$  for  $0 \leq t \leq 4/8$ .

Where  $h$  is the step length for  $x$  axis and  $k$  is the step size in time direction using Bender-Schmidt method

- C) Find inverse Laplace transform of  $\frac{1}{(s^2+16)(s^2+25)}$  using convolution theorem 8

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