

Time: 3 Hours

Total Marks: 80

- N.B. :** (1) Question No. 1 is compulsory.
 (2) Attempt any three questions out of the remaining five questions

Q.1 (a) Given the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 3 \\ 2 & 5 & 7 \end{bmatrix}$ find null space of A and rank of A. (5)

(b) A box contains 6 red balls and 4 blue balls. Three balls are drawn with replacement. Let X = number of red balls drawn. Find: (5)

- (i) Probability Distribution of X
 (ii) Cumulative Distribution function of X

(c) The weights (in kg) of 15 students are: 46, 48, 49, 51, 52, 52, 53, 54, 55, 56, 58, 60, 61, 95, 102 Identify the outlier if the outlier lies more than ± 2 standard deviations away from the mean. (5)

(d) Obtain the Hessian Matrix for the function $Z = 18x_1x_2 + 10x_1 - 34x_3 + 70x_1^2 + 40x_2^2 + 50x_3^2$ (5)

Q.2 (a) Find Singular Value of Decomposition of matrix $A = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 5 & 0 \end{bmatrix}$ (10)

(b) A company claims that the mean lifetime of a battery is 500 hours. A sample of 25 batteries gives:
 • Sample mean = 480 hours
 • Sample SD = 40 hours
 Test at 5% significance whether the population mean is different from 500 hours. (Given $t=2.145$ at 5% level of significance for a two-tailed test) (10)

Q.3 (a) Given the following data, plot a "less than" Ogive and use it to estimate the median score. (10)

Class Interval	Frequency
00-10	4
10-20	8
20-30	12
30-40	10
40-50	6

(b) Conduct a two tailed F Test on the following samples: (10)
 Sample 1: Variance = 109.63, sample size = 41.
 Sample 2: Variance = 65.99, sample size = 21
 (Given $F_{((40,20),0.025)}=2.287$, $F_{((40,20),0.975)}=0.4836$)

- Q.4 (a) Apply **Linear Discriminant Analysis (LDA)** for the following data of two classes: (10)

Class C1 – Apples (weight, colour intensity):

$$C1 = \{(150, 8), (160, 7), (155, 9), (165, 8)\}$$

Class C2 – Oranges (weight, colour intensity):

$$C2 = \{(120, 5), (130, 4), (125, 6), (135, 5)\}$$

Find:

- i. The **projection vector**
 - ii. The **discriminant function**
- (b) You are given the following data set of two features (variables) for 5 students: (10)

Student	A	B	C	D	E
Skill Score (X1)	90	80	70	60	50
Productivity Score (X2)	85	70	65	60	55

Compute:

- i. Covariance between X1 and X2
- ii. Correlation coefficient
- iii. Covariance matrix

- Q.5 (a) Minimize the function $f(x_1, x_2) = x_1x_2 - 8x_1 - 6x_2 + 6x_2 + x_1^2 + x_2^2$ (10)

subject to $2x_1 + x_2 = 12, x_1, x_2 \geq 0$

- (b) Find the minimizer of $f(x) = (x - 3)^2 + 2$ using bisection method in (0,6) with an accuracy of 0.1 (10)

- Q.6 Attempt any four (20)

- (a) Plot the graphs of the following functions over the interval $x \in [-3, 3]$ (5)

(a) e^{-x} (b) e^{-x^2}

- (b) Explain the curse of dimensionality with reference to: (5)

- i. Distance metrics
- ii. Volume scaling
- iii. Overfitting in machine learning

- (c) Write short notes on (5)

1. **Principal Component Regression (PCR)**

2. **Linear Discriminant Analysis**

- (d) Find column Space of $A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 6 & 9 \end{bmatrix}$ (5)

- (e) Explain and differentiate with examples: (5)

- i. **K-Nearest Neighbors (KNN)**
- ii. **Support Vector Classifier (SVC)**

- (f) Write short notes on Non gradient based optimization technique. (5)
