

(3 Hours)

[Total Marks: 80]

N.B. : 1) Question No. 1 is **Compulsory**.2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Fit a straight line to the following data (5)

X	1	2	3	4	5	6
Y	49	54	60	73	80	86

(b) Calculate Correlation coefficient between the variables x and y for the following data (5)

X	12	15	18	21	27
Y	2	4	6	8	12

(c) Let X be a continuous random variable with probability density function (5)

$$f(x) = \frac{x}{6} + k, \quad 0 \leq x \leq 3 \quad \text{Find } k \text{ and } (1 \leq x \leq 2).$$

(d) Find the line integral of $\vec{F} = x^2\vec{i} + xy\vec{j}$ along line OP where, (5)
 $O = (0,0)$ and $P = (1,1)$.**Q.2 (a)** A random variable x has the following probability function (6)

X	-2	-1	0	1	2	3
P(x)	0.1	k	0.2	2k	0.3	3k

Find i) k ii) $P(x > 2)$ iii) $E(X)$ **(b)** Prove that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is (6)
solenoidal and find the constants a,b,c if \vec{F} is irrotational.**(c)** Evaluate $\int_c \frac{z+6}{z^2-4} dz$ where c is (i) $|z| = 1$ (ii) $|z - 2| = 1$ (iii) $|z + 2| = 1$. (8)**Q.3 (a)** The average breaking strength of steel rods is specified to be 17.5 (in units of (6)
1000 kg) to test this sample of 14 rods tested & gave the following results: 15, 18, 16, 21, 19, 21, 17, 17, 15, 17, 20, 19, 17, 18. Is the result of the experiment significant?**(b)** Use Green's theorem to evaluate $\int_c (2x^2 - y^2) dx + (x^2 + y^2) dy$ where c is (6)
the boundary of the region enclosed by the lines $x = 0, y = 0, x = 2, y = 2$.**(c)** If height of 500 students are normally distributed with mean 68 inches and (8)
standard deviation 4 inches, Find the number of students having heights (i) greater than 72 inches (ii) between 65 and 71 inches (iii) less than 62 inches.

- Q.4 (a)** Use Gauss Divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} ds$ where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ and s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (6)

- (b)** Find the lines of regression for the following data to estimate y corresponding to $x = 155$ (6)

X	100	110	120	130	140	150	160	170	180	190
Y	45	51	54	61	66	70	74	78	85	89

- (c)** Find all possible Laurent's series expansion of the function $f(z) = \frac{5z+7}{(z+3)(z+2)}$ about $z = 0$ indicating region of convergence. (8)

- Q.5 (a)** The standard deviation from two random samples of sizes 9 and 13 are 1.99 and 1.9. Can the samples be regarded as drawn from normal population with same standard deviation? ($F_{(8,12)}(0.025) = 3.51, F_{(12,8)}(0.025) = 4.20$) (6)

- (b)** Using Stoke's Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = yi + zj + xk$ and C is the boundary of surface $x^2 + y^2 = 1 - z, z > 0$. (6)

- (c)** In an experiment on immunization of cattle from tuberculosis the following results were obtained (use 5% LOS) (8)

	Affected	Not Affected	Total
Inoculated	267	27	294
Not Inoculated	757	155	912
Total	1024	182	1206

Use Chi Square test to determine the efficiency of vaccine in preventing tuberculosis.

- Q.6 (a)** A bag contains 7 red balls and 3 black balls and another bag contains 4 red balls and 5 black balls. One ball is transferred from the first bag to the second bag then a ball is drawn from the second bag. If this ball happens to be red, Use Bayes' theorem to find the probability that a black ball was transferred. (6)

- (b)** A car hire firm has 2 cars which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the probability of days on which some demand is refused. (6)

- (c)** Show that $\vec{F} = (2xy + z)\hat{i} + (x^2 + 2yz^3)\hat{j} + (3y^2z^2 + x)\hat{k}$ is conservative. Find scalar potential such that $\vec{F} = \nabla\phi$ and hence, find the work done by \vec{F} in displacing a particle from (1,2,0) to (2,2,1). (8)
