

Time: (3 Hours)

[Total Marks: 80]

N.B.: 1) Question No. 1 is **Compulsory**.2) Answer **any THREE** questions from Q.2 to Q.6.

3) Figures to the right indicate full marks.

Q.1 (a) Calculate Correlation coefficient between the variables x and y for the following data (5)

X	1	2	4	5	3
Y	3	3	5	8	6

(b) A random variable x has the following probability function (5)

X	1	2	3	4	5
P(x)	3c	2c	2c	c	2c

Find i) C ii) $P(x < 3)$ iii) $E(X)$ iv) $V(X)$

(c) A random sample of 50 items gives the mean 6.2 and variance 10.24. Can it be regarded as drawn from a normal population with mean 5.4 at 5% level of significance? (5)

(d) Find $a, b,$ and c if $\vec{F} = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x - cy + 2z)\mathbf{k}$ is irrotational. (5)

Q.2 (a) Fit a straight line to the following data (6)

X	1	2	3	4	5
Y	5	8	3	9	6

(b) Find the work done in moving a particle in the force field $\vec{F} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$ along $x = t, y = t, z = t$ from $(0,0,0)$ to $(1,1,1)$. (6)(c) Find all possible Laurent's series expansion of the function $f(z) = \frac{3}{(z+2)(z+5)}$ about $z = 0$ indicating region of convergence. (8)Q.3 (a) Given: $2x + 6y = 90$, $9x + 3y = 130$ are regression lines and $\sigma_x^2 = 16$ then find (i) mean of X and Y (ii) correlation coefficient (r) (iii) σ_y^2 (6)(b) Use Green's theorem to evaluate $\int_c (x^2 - y) dx + (2y^2 + x) dy$ where c is the boundary of the region enclosed by $y = x^2$ and $y = 4$. (6)(c) Investigate the association between the darkness of eye colour in father and son from the following table using χ^2 -test (use 5% LOS) (8)

Colour of son's eyes	Colour of father's eyes			Total
		Dark	Not Dark	
Dark		48	90	138
Not Dark		80	782	862
Total		128	872	1000

Q.4 (a) Let X be a continuous random variable with probability density function $f(x) = ke^{-x}$, $x \geq 0$ Find k , mean and variance. (6)

(b) Following result were obtained from two samples each drawn from two different populations A and B (6)

Group	A	B
Sample Size	25	17
Sample SD	4	3

Test the hypothesis that variance of A is less than or equal to variance of B. Given $(F(0.05) = 2.24 \text{ for d.o.f. } 24 \text{ and } 16)$

(c) Show that $\vec{F} = (6xy + z^3)\mathbf{i} + (3x^2 - z)\mathbf{j} + (3xz^2 - y)\mathbf{k}$ is conservative. (8)
Find scalar potential such that $\vec{F} = \nabla\phi$ and hence, find the work done by in displacing a particle from (1,2,0) to (3,3,2) .

Q.5 (a) A fair coin is tossed till a head appears. What is the expectation of the number of tosses required? (6)

(b) Using Stoke's Theorem to evaluate $\int_c \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2\mathbf{i} + xy\mathbf{j}$ and c is the boundary of the rectangle $x = 0, y = 0, x = a, y = b$. (6)

(c) Evaluate $\int_c \frac{2z}{z^2-4} dz$ where c is (i) $|z - 2| = 1$ (ii) $|z + 2| = 1$. (8)

Q.6 (a) Four roads lead away from a jail. A prisoner trying to escape from the jail selected a road at random. If road A is selected, the probability of escaping is $1/8$, for road B it is $1/6$, for road C it is $1/4$ and for road D it is $9/10$. (6)

What is the probability that a prisoner will succeed in escaping from the jail

(b) Use Gauss Divergence theorem to evaluate $\iiint_s \vec{F} \cdot \hat{n} ds$ where (6)

$\vec{F} = 4x\mathbf{i} + 3y\mathbf{j} - 2z\mathbf{k}$ and s is the surface bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$

(c) Monthly salary X in a big organization is normally distributed with mean Rs 3000 and standard deviation of Rs 250. What should be the minimum salary of a worker in this organization, so that the probability that he belongs to top 5% workers? (8)