02/06/2025 SE CHEMICAL SEM-III C-SCHEME EM-III QP CODE: 10083689

(3 Hours) **Total Marks: 80**

Note: 1) Question No.**1** is **compulsory.**
2) Attempt **any THREE** from the remaining.
3) Figures to the right indicate full marks**.**

Q.1 **A)** Find the values of constants a,b,c and d if **5**
f(z) = $(x^2 + 2axy + by^2) + i(cx^2 + 2dxy + y^2)$ is analytic

B) Find the Eigen Value of $A^3 - 3A^2$ **5**
Where A = $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

C) Find the Laplace Transform of t sin at **5**

D) Find the Fourier series expansion for f(x) = x defined in (-1,1) **5**

Q.2 **A)** If L[f(t)] = $\frac{s}{s^2+s+4}$ find L[$e^{-3t}f(2t)$] **6**

B) Find the Fourier series expansion for f(x) = x defined in $(-\pi, \pi)$ with period 2π **6**

C) Find the analytic function f(z) with the real part u = $x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ **8**

Q.3 **A)** Show that the function u = $x^3 - 3xy^2$ is harmonic function. **6**
Hence find the corresponding analytic function and harmonic conjugate.

B) A string is stretched and fastened to two points distance L apart motion is started by displacing the string in the form u =$\alpha \sin(\frac{\pi x}{L})$ from which it is released at time t = 0. Show that the displacement of a point at a distance X from one end at time t is given by u(x,t) = $\alpha \sin\left(\frac{\pi x}{L}\right) \cos(\frac{\pi ct}{L})$ **6**

C) Obtain the Fourier series expansion of f(x) = $|x|$ where $-\pi \leq x \leq \pi$ **8**

Q.4 **A)** Find Laplace transform of $e^{-4t}\int_0^t usin3u\, du$ **6**

B) Find Inverse Laplace transform of $\frac{2s+3}{s^2+2s+2}$ **6**

C) Verify Cayley – Hamilton theorem for the matrix A and hence find A^{-1} & A^4 **8**
where A = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

Q.5 **A)** Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16\frac{\partial u}{\partial t} = 0$,$0 \leq x \leq 1$ **6**
subject to the condition u(0,t) = 0,u(1,t) = 100t , u(x,0) = 0, h = $\frac{1}{4}$
for one –time step.

B) Find the inverse Laplace transform of log $\left(\frac{s+a}{s+b}\right)$ **6**

C) Show that the matrix A = $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 7 \end{bmatrix}$ is diagonalizable. **8**
Find transforming matrix and diagonal Matrix.

Q.6 **A)** Evaluate $\int_0^\infty e^{-3t}\, t\, sint\, dt$ using Laplace transform. **6**

B) Find the solution $u_t = u_{xx}$ subject to u(0,t) = 0,u (5,t) = 0,u (x,0) = $x^2\,(25 - x^2)$ **6**
using Schmidt method taking h = 1 up to 3 seconds.

C) Find the inverse Laplace transform of $\frac{s}{(s^2+1)^2}$ using convolution theorem. **8**

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