

10 NOV 2025 SE SEM-III C SCHEME CHEMICAL EM-III QP CODE: 10096312

(3 Hours)

Total Marks :80

Note: 1) Question No.1 is compulsory
2) Attempt any Three from the remaining

Q.1.

- A) Find $L \left\{ \int_0^t u^n e^{6u} du \right\}$ 5
- B) Prove that $f(z) = e^x(\cos y + i \sin y)$ is analytic everywhere. Hence find $f'(z)$ 5
- C) Find half range sine series of $f(x) = x^2$ in $(0, \pi)$ 5
- D) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ then Find an eigen value of 5
- i) A
- ii) $\text{Adj}(A)$
- iii) $A^2 - 2A + I$

Q.2.

- A) If $L[f(t)] = \frac{1}{s^2+1}$ then Find $L[te^t f(t)]$ 6
- B) Find Fourier series for $f(x) = x^3$, if $0 < x < 2\pi$ and $f(x + 2\pi) = f(x)$ 6
- C) Find analytic function $f(z)$ in terms of z where $u = x^2 + y^2 + 5x + y + 2$ 8

Q.3.

- A) A string is stretched and fastened to two points distance l apart. Motion is started by displacing the string in the form $y = a \sin(\pi x / l)$ from which it is released at time $t=0$. Show that the displacement of a point at a distance x from one end at time t is given by $y = a \sin(\pi x / l) \cos(\pi ct / l)$ 6
- B) Prove that $u = y^3 - 3x^2y$ is harmonic function hence find its harmonic conjugate function 6
- C) Find the Fourier Series for $f(x) = |x|$ in $(-\pi, \pi)$ where 8

Q.4.

- A) Evaluate $\int_0^\infty \left[\frac{\sin at}{t} \right] dt$ 6
- B) Find Inverse Laplace transform of $\frac{s}{(s-1)(s-2)(s-3)}$ 6
- C) Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Diagonalizable? If so find the Diagonal form of A and transforming matrix of A 8

Q.5.

A) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ then verify Caley Hamilton theorem and hence 6

Compute: $A^5 + A^4 - 18A^3 - 40A^2$

B) Solve by Crank-Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} - 16 \frac{\partial u}{\partial t} = 0,$ 6

$0 \leq x \leq 1$ subject to the condition $u(0, t) = 0, u(1, t) = 100t,$

$u(x, 0) = 0$ $h = 0.25$ for one-time step

C) Find inverse Laplace transform of (i) $\log \left[\frac{s+1}{s+5} \right]$ (ii) $\frac{s-4}{(s+6)^2}$ 8

Q.6.

A) Find the Laplace Transform of $e^{-t} \int_0^t \sin(u) \cos(u) du$ 6

B) Find the solution of 6

$$4 \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0, 0 < x < 8, \quad u(x, 0) = 4x - \frac{1}{2}x^2, u(0, t) = 0, u(8, t) = 0$$

Taking $h = 1, k = \frac{1}{8}$ for $0 \leq t \leq 4/8$

Where h is the step length for x axis and k is the step size in time direction using Bender-Schmidt method

C) Find inverse Laplace transform of $\frac{1}{(s^2+16)(s^2+25)}$ using convolution theorem 8
