

Time: 3 Hours

Total Marks: 60

N.B.

1. Question No. 1 is Compulsory.
2. Attempt any **Three** Questions from remaining **Five** Questions
3. Assume Suitable Data if needed and Justify the Same
4. Figures to the right indicate full marks.

Que.1 Solve any **FIVE**

- a) Explain Types of correlations [03]
- b) Find the Eigen values of 3×3 matrix are -1,2 if the determinant of matrix is 4 find third Eigen value. [03]
- c) Solve $y_{n+2} - 6y_{n+1} + 9y_n = 0$ [03]
- d) Find Laplace Transform of $\sin^2 t + e^{6t} + \sin 2t - 5\cos t$ [03]
- e) Evaluate: $\frac{\Delta^2}{E} x^2$, consider $h=1$ [03]
- f) What is the key difference between an initial value problem (IVP) and a boundary value problem (BVP)? [03]

Que.2

- a) Measuring pressure drop ΔP (kPa) over pipe length L (m) [05]

L (m)	1	2	3	4	5
ΔP (kPa)	2.1	4.3	6.4	8.6	10.7

Fit the data in $\Delta P = a + bL$ and predict ΔP at $L = 3.6$ m.

- b) Find the Eigen Value and Eigen Vector of following matrix [06]

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- c) Explain Discrete and Continuous random variables with examples [4]

Que.3

- a) A dynamic model for the flow through cylindrical tank, [6]

$$\frac{dh}{dt} = \frac{F_0 - F}{\frac{\pi}{4} D^2}$$

Where, D is diameter of tank F_0 is the inlet flow rate, F is the outlet flow rate and h is the level in the tank. If the tank dimensions are as given below, find the liquid level in the tank with respect to time. Take step size of 5 second and show calculations till 10 seconds. Use Euler's method

Data : Diameter of tank = 1 m, height of tank = 2 m, $F = 0.04\sqrt{h}$ in m^3/s where h is in m, $F_0 = 0.06 m^3/s$ at $t=0$ sec, $h=1$ m.

- b) The pressure (bar) and gas solubility (mole fraction) of CO_2 in water are measured. Find Karl Pearson's correlation coefficient. [05]

Pressure	100	200	300	400	500
CO_2	30	40	50	60	70

- c) What is the defining characteristic that separates higher-order RK methods from Euler's method? [4]

Que.4

- a) Solve $\frac{\partial p}{\partial t} = \frac{\partial^2 p}{\partial y^2}$ subject to the condition $p(y,0)=0$, $p(0,t)=0$, $p(1,t)=80t$ with $k=1/4$, $h=1/2$ for a one time step using Crank Nicholson method. [06]
- b) Solve $y_{n+1} - 2y_n = \cos 2n$ [04]
- c) Check whether the following matrix A is diagonalizable? [05]

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 3 & 2 \end{bmatrix}$$

If so, find the diagonal form of A and transforming matrix of A.

Que.5

- a) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, given $u(0,t)=0$, $u(4,t)=0$, $u(x,0)=x(4-x)$ assuming $h=k=1$ find values of u up to $t=5$ using Bender Schmidt method. [06]
- b) State First Shifting Theorem and Solve for $L(e^{-t} \cosh 4t)$ [05]
- c) Solve the following Difference equation [04]
- $$y_{n+2} - 7y_{n+1} + 10y_n = 3^n + 4^n$$

Que.6

- a) A chemical reaction is carried out in batch reactor and it has been found that concentration of reactant changes as per the equation given below. [05]

$$\frac{dC_A}{dt} = \frac{C_A}{1 + 0.5C_A^{1.8}}$$

If initial concentration (at $t=0$) is 0.8 then find concentration of reactant at $t=1$ with step size, $h=0.5$ using Runge kutta second order method.

- b) Solve $y_{n+2} - 6y_{n+1} + 9y_n = 0$ given $y_0 = 1$, $y_1 = 0$ [05]
- c) Find the Inverse of Laplace transform of [05]

$$L^{-1} \left(\frac{4}{s^6} - \frac{2}{s^{10}} + \frac{2}{s^2 - 9} + \frac{3s}{s^2 + 25} \right)$$
