(REVISED COURSE)

(3 Hours) **[Total Marks: 80**

N.B. :

1) **Question 1 is compulsory**. Answer any **three** more from the remaining questions.
2) Assume data if necessary and **specify the assumptions** clearly.
3) Draw neat sketches wherever required.
4) Answers to the sub-questions of an individual question should be grouped and written together i.e. one below the other.

1. (a) Write the dual of the following LP: **[05]**

$$\begin{aligned} Max \;\; z &= x_1 + 2x_2 - 2x_3 \\ s.t. \;\; -x_1 + 2x_2 - x_3 &\geq -8 \\ 2x_1 - x_2 + 2x_3 &= 10 \\ -x_1 - x_2 + x_3 &\geq -10 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

(b) The following table represents the additional features that a racing car can have, to increase its top speed along with the cost of each feature : **[05]**

	Features					
	1	2	3	4	5	6
Cost	10.2	6.0	23.0	11.1	9.8	31.6
Speed	8	3	15	7	10	12

The cost is in thousands of dollars, and the additional speed that can be gained with each feature is in kmph. Formulate the problem to maximize the speed gain subject to a budget constraint of \$35000/ =

(c) The college canteen consumes $50\,kg$ of potatoes everyday. The current policy is to purchase potatoes at a rate of $Rs.\,1600$ for a bag of $100\,kg$, and keep it in the store. It costs $Rs.\,50$ to place an order, and $Rs.\,0.02$ per day to store one kg of potatoes. It takes one day for the potatoes to be delivered after the order is placed. Determine the optimal policy. **[05]**

(d) Customers arrive randomly at a check-out counter at an average rate of 20 per hour. What is the probability that at least two customers are waiting in line for service? **[05]**

[TURN OVER

2. (a) Use Dijkstra's algorithm to find the shortest path from city A to city D in the following network: **[10]**

From city	*To city*	*distance in km*
A	B	05
A	C	20
A	E	05
B	C	12
B	E	03
D	B	04
D	E	06
E	C	02

(b) Solve the following LP using the Revised Simplex method: **[10]**

$$Max \;\; z = x_1 + x_2 + x_3$$
$$s.t.\; 2x_1 + x_2 + 2x_3 \leq 8$$
$$x_1 + 2x_2 + x_3 \leq 12$$
$$x_1, x_2, x_3 \geq 0$$

3. Solve the following transportation problem using Vogel's Approximation: **[20]**

Source	*Dest. #1*	*Dest. #2*	*Dest. #3*	*Cap.*
1	22	18	24	365
2	16	22	18	545
3	24	24	20	240
Demand	420	380	225	

The entries represent cost of transportation in *rupees/unit* from each supply node to each destination node.

4. (a) Solve the following IP using Branch and Bound Technique: **[10]**

$$Max \;\; z = 2x_1 + 3x_2$$
$$s.t.\; x_1 + 3x_2 \leq 5$$
$$3x_1 + x_2 \leq 10$$
$$x_1, x_2 \geq 0 \;\; and \; integer$$

(b) Consider the inventory situation in which orders are filled at a constant rate of r *units/time*. Demand is constant at d *units/time*. Consumption occurs during replenishment period also, and so it is necessary that $r > d$. The setup cost is k and the holding cost is h *per unit per time*. If the order size is Q and shortage is not allowed, obtain expressions for the following: **[10]**

- maximum inventory level.
- the total cost per unit time.
- the economic order quantity.

Show how the *EOQ* formula can be obtained for instantaneous replenishment from the above expression.

5. (a) A factory manufactures three products, which require three resources- labor, materials, and technical expertise. The unit profits on these products are \$10, \$6, *and* \$4, respectively. There are 100 hr of labor, 600 kg of material, and 300 hr of technical expertise available per day. In order to determine the optimal product mix, the following LP model is formulated and solved: **[10]**

$$\begin{aligned} Max\ z &= 10x_1 + 6x_2 + 4x_3 \\ s.t.\ x_1 + x_2 + x_3 &\leq 100 \\ 10x_1 + 4x_2 + 5x_3 &\leq 600 \\ 2x_1 + 2x_2 + 6x_3 &\leq 300 \\ x_1 \geq 0, x_2 &\geq 0, x_3 \geq 0 \end{aligned}$$

where x_1, x_2, x_3 arev the daily production levels of products 1, 2, 3, respectively. What is the worth of each resource? What are the ranges of values for the resources that will ensure the feasibility of the optimum obtained?

(b) The following table represents the pay-off matrix for player A: **[10]**

A's strategy	*B's strategy*		
	*B*1	*B*2	*B*3
*A*1	2	1	2
*A*2	4	3	2
*A*3	1	2	6

Calculate the optimal strategy for both players.

6. (a) A business executive must make the four round trips listed below between the head office in Dallas and a branch office in Atlanta. **[10]**

The price of a round trip ticket from Dallas is \$400. A discount of 25% is granted if the dates of arrival and departure of a ticket span a weekend(Saturday and Sunday). If the stay in Atlanta lasts more than 21 days, the discount is increased to 30%. A one-way ticket between Dallas and Atlanta(either direction) costs \$250. How should the execytive purchase the tickets?

Departure date from Dallas	*Return date to Dallas*
Monday, June 3	*Friday, June* 7
Monday, June 10	*Wednesday, June* 12
Monday, June 17	*Friday, June* 21
Tuesday, June 25	*Friday, June* 28

(b) Write a note on the Traveling Salesman Problem and the methods of solution. **[10]**

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