(REVISED COURSE)

(3 Hours) [Total Marks: 80

N.B. :

1) **Question 1 is compulsory**. Answer any **three** more from the remaining questions.
2) Assume data if necessary and **specify the assumptions** clearly.
3) Draw neat sketches wherever required.
4) Answers to the sub-questions of an individual question should be grouped and written together i.e. one below the other.

1. (a) The flow of a particular fluid is described by its velocity components as: **[05]**

$$u = -\frac{V_0}{l}x, \quad v = -\frac{V_0}{l}y$$

where V_0 *and* l are constants. Derive an expression to calculate the rate of change of the density of the fluid $\rho(x, y, z, t)$ with respect to time, along a flowing fluid particle.

(b) Consider the following equation: **[05]**

$$\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} = 0$$

Derive an expression to solve the equation numerically using an implicit scheme.

(c) Consider the following function: **[05]**

$$f(x, y) = exp(-x^2) + exp(-y^2)$$

Use backward differences to calculate:

$$\frac{\partial f(x, y)}{\partial x} \; and \; \frac{\partial f(x, y)}{\partial y}$$

at $x = 0.1,\ and\ y = 0.1$. Also calculate the error. Take

$$\Delta x = 0.05 \; and \; \Delta y = 0.05$$

(d) Write a short note on the upwind scheme. **[05]**

2. (a) An insulated metal rod has an initial temperature profile given by: **[10]**

$$T(x, 0) = 20^o C$$

At time $t = 0$, a hot reservoir at a temperature of $T = 200^o C$ is brought into contact with the left end of the rod. Simultaneously the right end is exposed to a hot reservoir at a temperature of $300^o C$. The length of the rod is 1 m, and for the metal $\alpha = 1.0 \times 10^{-5}\ m^2/sec$. Using the finite difference FTCS scheme, calculate the temperature across the rod for the next 1600 sec. Assume $r = 0.2$ *and* $\Delta x = 0.2\,m$.

(b) Solve the following equation using the weighted residual method: **[10]**

$$\frac{d^2u}{dx^2} = -(x + 1)$$

with $u = 0\ at\ x = 0\ \ and\ \frac{du}{dx} = 1\ at\ x = 2$ Use the trial function $u = a_1 x + a_2 x^2$

3. Consider the steady, one dimensional heat conduction, in an insulated metal rod, of length 0.8 m. The ends of the rod are maintained at constant temperatures of 200^oC and 400^oC respectively. Applying the finite volume method on eight control volumes of equal length, determine the temperature profile across the rod. Thermal conductivity of the metal is $k = 1000\ W/m.K$ and the cross-sectional area of the rod is $A = 0.01\ m^2$ **[20]**

4. (a) Consider the following equation in the domain $0 \leq x \leq L$: **[10]**

$$\frac{d^2u}{dx^2} + q = 0$$

The boundary conditions are:

$$u(0) = u_0\ and\ \ \frac{du}{dx} = 0\ \ at\ x = L$$

Using the linear elements:

$$N_1 = \left(1 - \frac{x}{l}\right),\ \ and\ \ N_2 = \frac{x}{l}$$

and applying the weak form of the Galerkin method, develop a numerical scheme to solve the equation.

(b) The governing equation for a fully developed steady laminar flow of a Newtonian viscous fluid on an inclined flat surface is given by: **[10]**

$$\mu\frac{d^2v}{dx^2} + \rho g \cos\theta = 0$$

where

$$\mu = coefficient\ of\ viscosity$$
$$v = fluid\ velocity$$
$$\rho = density$$
$$g = acceleration\ due\ to\ gravity$$
$$\theta = angle\ of\ the\ inclined\ surface\ with\ the\ vertical$$

The boundary conditions are given by:

$$\left(\frac{dv}{dx}\right)_{x=0} = 0$$

$$v(w) = 0$$

Here w is the thickness of the laminar film, and x is measured from the free surface of the film. Find the velocity distribution $v(x)$ using the weighted residual method.

5. (a) Consider the following function: **[10]**

$$y(x) = 1 + \cos(0.1\pi x) + \sin(0.1\pi x)$$

Use quadratic interpolation to approximate the given function, at three points, within the domain $0 \leq x \leq 1$, and calculate the error.

(b) Consider the cooling of a circular fin by convective heat transfer along its length. The cylindrical fin has a uniform cross-sectional area A, and perimeter P. The base of the fin is at a temperature of $T_B = 300\,^oC$, and the free end is insulated. The fin is exposed to an ambient temperature $T_A = 30\,^oC$. Calculate the temperature distribution along the fin, using a trial function: **[10]**

$$T(x) = c_0 + c_1 x + c_2 x^2$$

Data:

$$n^2 = \frac{hP}{kA} = 25\,m^{-2} \quad L = 1\,m$$

6. (a) Diffusion and reaction take place in a pore of length $1\,mm$. The rate constant of the first order reaction is $k = 10^{-3}\,s^{-1}$, and the effective diffusivity of the reacting species is $D = 10^{-9}\,m^2/s$. Dividing the pore into five equal parts obtain the concentration profile along its lenfth, using central differencing scheme. The concentration at the mouth of the pore is $C(0) = 1\,mol/m^3$. The governing equation is given by: **[10]**

$$\frac{d^2C}{dx^2} - \frac{k}{D}C = 0$$

with the boundary conditions:

$$C(0) = 1 \quad and \quad at\ x = 1\,mm, \frac{dC}{dx} = 0$$

(b) A company produces a perishable product in a factory at $x = 0$, and sells it along the distribution route $x > 0$. The selling price of the product, p, is a function of the length of time after it was produced, t, and the location at which it is sold, x, i.e. $p = p(t, x)$. At a given location the price decreases in time at a rate of $-8\$/hr$. In addition, because of shipping costs, the price increases with distance from the factory at a rate of $0.1\$/km$. If the manufacturer wants to sell the product at the same price of $100 everywhere, determine how fast he must travel along the distribution route. **[10]**

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