Q. P. Code: 34333

(2 ½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Attempt All(Each of 5Marks) (15M)

(a) Multiple Choice Questions.

i) The set of all linear combinations of vectors $v_1, v_2, \ldots, v_n$ is called the __________ of the vectors.
a) Convex b) concave c) span d) combination.

ii) Nullity of T is the dimension of ______ of T.
a) Kernel b) Image c) Rank d) none of the above.

iii) $|A - \lambda I| = 0$ is called ________ equation.
a) Quadratic b) characteristics c) cubic d) Null.

iv) In GF (2), 1+1+0+1 = ___________.
a) 0 b) 1 c) 3 d) 2.

v) For any homogenous system _________ is a trivial solution.
a) Zero b) non zero c) one d) none of the above.

(b) Fill in the blanks.
(Spare, Unique, Unit, $\sqrt{45}$, Inner product)

i) A vector whose norm is one is called _________ vector.

ii) A vector space together with inner product is called _______ space.

iii) If most of the element of a matrix have zero value is called ______ matrix.

iv) The absolute value of 3+6i = _________.

v) Inverse of a matrix is __________.

(c) Define.

i) Dot product.
ii) Gatois field.
iii) Eigen Value.
iv) Orthogonal Complement.
v) Dimension.

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Q. 2 Attempt the following (Any THREE) (15M)

(a) Find the Square root of 21 – 20i, where i= $\sqrt{-1}$

(b) Consider the following system of equation and find the nature of solution without solving it.

i) $x_1 + x_2 = 4$
$2x_1 + 2x_2 = 8$

ii) $x_1 + x_2 = 3$
$x_1 - x_2 = -1$

(c) Solve the following system by backward substitution method

$x_1 - 3x_2 - 2x_3 = 7$
$2x_2 + 4x_3 = 4$
$10x_3 = 20$

(d) Let W_1 and W_2 are two subspaces of V then prove that $W_1 \cap W_2$ is also a subspace of V where V is a vector space on IR.

(e) Write a python Program for rotating a complex number Z = 2+3i by 180^o

(f) Which of the following is a set of generators of IR^3

i) {(4, 0,0), (0,0,2)}

ii) {(1, 0,0), (0,1,0), (0,0,1)}

Q. 3 Attempt the following (Any THREE) (15M)

(a) Find the null space of matrix $\begin{bmatrix} 1 & 5 & 6 \\ 2 & 6 & 8 \\ 3 & 4 & 7 \end{bmatrix}$

(b) Let f: U→ V is a linear transformation then show that kerf = {0} iff f is injective.

(c) Find the co-ordinate representation of vector v = (0, 0, 0,1) in terms of the vectors [1,1,0, 1], [0,1,0, 1] and [1,1,0, 0] in GF (2).

(d) Find the angle between the two vectors $a =$ (2,3,4) and b=(1, −4,3) in IR^3.

(e) Consider Subspace $U_1\{(x, y, w, z) : x - y = 0\}$ and $U_2\{(x, y, w, z) : x = w, y = z\}$ Find a basis and dimension of

i) U_1 ii) U_2 iii) $U_1 \cap U_2$.

(f) If V and W are two subsets of a vector space V such that U is a subset of W then show that W^0 is a subset of U^0 where U^0, W^0 are annihilator of U and W respectively.

Q. 4 Attempt the following (Any THREE) (15)

(a) Let u and v are orthogonal vectors then prove that for scalars a,b.
$||au + bv||^2 = a^2||u||^2 + b^2||v||^2$

(b) Explain Internet Worm.

(c) Write a program in python to final gcd (240,24)

(d) Solve the following system by Gaussian elimination method.

y –z = 3

-2x + 4y -z = 1

-2x + 5y – 4z = -2

(e) Find the orthonormal basis for subspace IR^4 whose generators are $v_1 = (1,1,1,1), v_2 = (1,2,4,5), v_3 = (1,-3,-4,-2)$ Using Gram Schmidt orthogonali sation Method.

(f) Let a = (3,0), b = (2,1) find vector in span $\{a\}$ that is closet to b is $b^{||a}$ and distance $||b^{\perp a}||$.

Q. 5 Attempt the following (Any THREE) (15)

(a) Let T : $|R^3 \rightarrow |R^2$ be a linear map defined by f (x,y,z) = (x+2y-z, x+y-2z) Verify Rank T + Nullity T = 3.

(b) Fill the table.

Vector space	Basis	Dimension
{0}		
IR^2	{(1,0),(0,1)}	
$P_2(x)$		3
$M_2(IR)$		4
IR	{1}	

(c) Find eigen values and eigen vectors of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

(d) Let S be a subset of vector space V. Prove that $S^{\perp}$ is a subspace of V.

(e) Check whether the following set {(1,1,0), (0,1,1), (1,1,1)} is linearly Independent or not.
