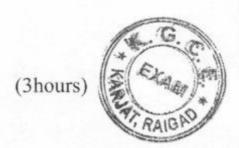
Sem-II/mech & Brod/CBGS/A-Maths:III/May 17

Q. P. Code: 11700



[Total marks: 80]



- (2) Answer any Three from remaining
- (3) Figures to the right indicate full marks
- 1. (a) State Cauchy Reimann equation in polar form. Find p if

$$f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$$
 is analytic.

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- (b) Find Laplace transform of sin 2 t. cos 3 t.
- (c) Prove $\{\sin n x\}$, $n = 1, 2, 3, \dots$ is orthogonal w.r.t. $(0, 2\pi)$.
- (d) Evaluate $\int_{1+i}^{2+4i} (x^2 + ixy) dz$ along the curve x = t, $y = t^2$
- 2. (a) Using Laplace transform, solve the differential equation,

$$\frac{dx}{dt} + 3x = 2 + e^{-t}$$
, with $x(0) = 1$.

(b) Evaluate
$$\oint_C \frac{z+1}{z^3 - 2z^2} dz$$
 where $C : |z| = 1$.

(c) Obtain the Taylor's and Laurent series which represent the function

$$\frac{z^2-1}{(z+3)(z+4)}$$
 in the regions, (i) $|z| < 3$ (ii) $3 < |z| < 4$ (iii) $|z| > 4$.

3. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 32 \frac{\partial u}{\partial t} = 0$ by Bender –Schmidt method, given u(0,t) = u(x,0) = 0, u(1,t) = t, taking h = 0.25.

(b) Evaluate-
$$\int_{0}^{\infty} t e^{-3t} \sin t dt$$

(c) Obtain Half Range Sine Series of $f(x) = x(\pi - x)$ in $(0, \pi)$.

Hence, evaluate
$$-\sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)^3}$$
.

[TURN OVER]

- 4. (a) Find the orthogonal trajectory of the family of curves $x^3y xy^3 = 0$.
 - (b) Find Fourier series of f(x) = |x| in (-3, 3).
 - (c) Find the inverse Laplace transform of the following:-
 - (i) $\cot^{-1} s$ (ii) $\frac{8e^{-3s}}{s^2 + 4}$
- 5. (a) Solve by Crank Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} \frac{\partial u}{\partial t} = 0$,

u(0,t) = u(1,t) = 0, u(x,0) = 100x(1-x)taking h = 0.25 for one time step.

(b) Using convolution theorem find the inverse Laplace transform of

 $\frac{s}{(s^2+1)(s^2+4)}$

- (c) Find bilinear transformation which maps the points z = 1, i, -1 onto the points w = i, 0, -1. Hence, find the image of $|z| \le 1$ onto the w-plane.
- 6. (a) Using Residue theorem, evaluate, $\int_{0}^{\infty} \frac{dx}{x^2 + 1}$.
 - (b) Obtain Complex form of Fourier series for $f(x) = e^{ax}$ over $-\pi < x < \pi$. 6
 - (c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under boundary condition u(0,t) = u(l,t) = 0, u(x,0) = x, l being the length of rod.