Sem - III | App. maths - III / MECH and PROD

14-05-15

Q.P. Code : 4755

K.G.C.E. EXAM KARJAT, RAIGAD

(3 Hours) **[Total Marks : 80**

N.B. : (1) Question No. **1** is **compulsory.**

(2) Attempt **any three** questions out of **remaining five** questions.

1 (a) Find the Laplace transform of $te^{-t}\cosh 2t$ 5

(b) Find the fixed points of $w = \frac{3z-4}{z-1}$. Also express it in the normal form $\frac{1}{w-\alpha} = \frac{1}{z-\alpha} + \lambda$ where λ is a constant and α is the fixed point. Is this transformation parabolic? 5

(c) Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path i) $y=x$, ii) $y=x^2$ 5

(d) Prove that $f_1(x)=1$, $f_2(x)=x$, $f_3(x) = \frac{3x^2-1}{2}$ are orthogonal over $(-1,1)$ 5

2. (a) Find inverse Laplace transform of $\frac{2s}{s^4+4}$ 6

(b) Find the image of the triangular region whose vertices are i, 1+i, 1-i under the transformation $w = z+4-2i$. Draw the sketch. 6

(c) Obtain fourier expansion of $f(x) = |\cos x|$ in $(-\pi, \pi)$. 8

3. (a) Obtain complex form of fourier series for $f(x)=\cosh 2x + \sinh 2x$ in $(-2,2)$. 6

(b) Using Carnk-Nicholson simplified formula solve $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ given $u(0,t)=0, u(4,t)=0, u(x,0) = \frac{x}{3}(16-x^2)$ find uij for i=0,1,2,3,4 and j=0, 1,2 6

(c) Solve the equation $y + \int_0^t y\,dt = 1-e^{-t}$ 8

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4. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+3\sin\theta}$ 6

(b) Find half - range cosine series for $f(x)=e^x$, $0<x<1$ 6

(c) Obtain two distinct Laurent's series for $f(z) = \frac{2z-3}{z^2-4z-3}$ in powers of (z-4) indicating the regions of convergence. 8

5. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial t} = 0$ by Bender - Schmidt method, given u(0, t) = 0, u(4, t) = 0, u(x,0)= x(4-x). Assume h=1 and find the values of u upto t = 5 6

(b) Find the Laplace transform of $e^{-4t}\int_0^t u\sin 3u\,du$ 6

(c) Evaluate $\int_C \frac{z+3}{z^2+2z+5}\,dz$ where C is the circle i) $|z| = 1$, ii) $|z+1-i|=2$ 8

6. (a) Find inverse Laplace transform of $\frac{s}{(s^2-a^2)^2}$ by using convolution theorem. 6

(b) Find an analytic function f(z) = u+iv where $u+v=e^x$ (cosy + siny) 6

(c) Solve the equation $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial^2 x^2}$ for the conduction of heat along a rod of length l subject to following conditions 8

(i) u is not infinity for $t \to \infty$

(ii) $\frac{\partial u}{\partial x} = 0$ for x=0 and $x=l$ for any time t

(iii) $u=lx-x^2$ for t=0 between x = 0 and $x=l$
