Paper / Subject Code: 51901 / APPLIED MATHEMATICS- III

S.E. SEM III / PROD / CHOICE BASED / NOV 2019 / 14.11.2019

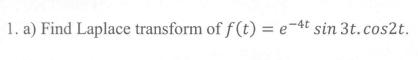
(3hours)

[Total marks: 80]

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N.B. 1) Question No. 1 is compulsory.

- 2) Answer any Three from remaining
- 3) Figures to the right indicate full marks





- b) Show that the set of functions f(x) = 1, g(x) = x are orthogonal on (-1,1). Determine the constants a and b such that the function $h(x) = -1 + ax + bx^2$ is orthogonal to both f(x) and g(x).
- c) Evaluate $\int_C (z^2 2\bar{z} + 1) dz$ where C is the circle |z| = 1.
- d) Compute the Spearman's Rank correlation coefficient *R* and Karl Pearson's correlation coefficient *r* from the following data,

X	12	17	22	27	32
y	113	119	117	115	121

- 2. a) Using Laplace transform, evaluate $\int_0^\infty e^{-t} \int_0^t \frac{\sin u}{u} du dt.$ 6
 - b) Find an analytic function f(z) = u + iv, if $u = e^{-x} \{ (x^2 y^2) \cos y + 2xy \sin y \}.$
 - c) Obtain Fourier series of $f(x) = x^2$ in $(0,2\pi)$. Hence, deduce that 8 $\frac{\pi^2}{12} = \frac{1}{1^2} \frac{1}{2^2} + \frac{1}{3^2} \frac{1}{4^2} + + \cdots$
- 3. a) Using Bender Schmidt method, solve $\frac{\partial^2 u}{\partial x^2} \frac{\partial u}{\partial t} = 0$, subject to the conditions,

$$u(0,t) = 0$$
, $u(4,t) = 0$, $u(x,0) = x^2(16 - x^2)$ taking $h = 1$, for 3 minutes. 6

b) Using convolution theorem, find the inverse Laplace transform of

$$F(s) = \frac{s^2 + s}{(s^2 + 1)(s^2 + 2s + 2)}$$

c) Using Residue theorem, evaluate

i)
$$\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta}$$
 ii) $\int_C \frac{z^2}{(z+1)^2(z-2)} dz$, $C: |z| = 1.5$

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- 4. a) Solve by Crank Nicholson simplified formula $\frac{\partial^2 u}{\partial x^2} 16 \frac{\partial u}{\partial t} = 0$, u(0,t) = 0, u(1,t) = 200t, u(x,0) = 0 taking h = 0.25 for one-time step.
 - b) Obtain the Laurent series which represent the function

$$f(z) = \frac{4z+3}{z(z-3)(z+2)}$$
 in the regions, i) $2 < |z| < 3$ ii) $|z| > 3$

- c) Solve $(D^2 3D + 2)y = 4e^{2t}$ with y(0) = -3 and y'(0) = 5 where $D \equiv \frac{d}{dt}$
- 5. a) Find the bilinear transformation under which 1, i, -1 from the z-plane are mapped onto $0,1,\infty$ of w-plane. 6

b) Find the Laplace transform of
$$f(t) = \begin{cases} t, & 0 < t < \pi \\ \pi - t, & \pi < t < 2\pi \end{cases} \text{ and } f(t + 2\pi) = f(t).$$

- c) Obtain half range Fourier cosine series of f(x) = x, 0 < x < 2. Using Parseval's identity, deduce that -8 $\frac{\pi^4}{96} = \frac{1}{14} + \frac{1}{34} + \frac{1}{54} + \cdots$
- 6. a) Using contour integration, evaluate: 6 $\int_{0}^{\infty} \frac{x^2 + x + 2}{x^4 + 10x^2 + 9} dx$
 - b) Using least square method, fit a parabola, $y = a + bx + cx^2$ to the following data,

x	-2) -1	0	1	2
v	-3.150	-1.390	0.620	2.880	5.378

c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ under the boundary conditions u(0,t) = 0, u(l,t) = 0, u(x,0) = x, (0 < x < l), lbeing the length of the rod.