Duration: 3 hours **Total Marks: 80**

Note: 1. Question No. 1 is compulsory

2. Attempt any Three questions from the remaining Five questions.

3. Use of statistical table is allowed

Q.1.a. If the product of eigen values of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16 , find the third eigen value (5)

b If X is a Poisson variate such that P(X=1) = P(X=2). Find E(X^2) (5)

c Find the angle between the normals to the surface x y = z^2, at the points (1 ,4 ,2) & (-3 ,-3 ,3) (5)

d A random sample of 50 items given the mean 6.2 and variance 10.24. Can it be regarded as drawn from a Normal population with mean 5.4? (5)

Q.2.a A vector field is given by $\bar{F}$ = (y Sin z – Sin x) i + (x Sin z + 2yz) j + (x y Cos z + y^2) k , prove that it is irrotational and hence , find its scalar potential (6)

b Following results were obtained from 2 samples each drawn from the different population A & B. (6)

Population	A	B
Sample size	25	17
Sample SD	4	3

Test the hypothesis that variance of A is less than or equal to variance of B

c Show that the matrix A = $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal form D and the diagonalising marix M (8)

Q.3.a In a distribution exactly normal, 7% of items are under 35 and 89% are under 63. What are the mean and standard deviation? (6)

b Using the method of Lagrange's multipliers , solve the following NLPP (6)

Optimise Z = 12 x_1 + 8x_2 + 6x_3 – x_1^2 - x_2^2 – x_3^2 – 23

Subject to $x_1 + x_2 + x_3 = 10$

$X_1 , x_2 , x_3 \geq 10$

c Evaluate $\int_C \bar{F}.d\bar{r}$ where $\bar{F}$ = yz i + xz j +xy k and C is the portion of the curve $\bar{r}$ = a Cos t i +b Sin t j + ct k from t = 0 to t= $\pi/4$ (8)

Q.4.a The height of 6 randomly chosen sailors are in inches: 63, 65 , 68, 69, 71, 72. The heights of 10 randomly chosen soldiers are: 61, 62, 65, 66, 69, 69, 70, 71, 72, 73. Discuss in the light that these data throw on the suggestion that the soldiers on an average are taller than the sailors. (6)

b Obtain the rank correlation from the following data (6)

X	10	12	18	18	15	40
Y	12	18	25	25	50	25

c If A = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find e^{At} (8)

Q.5.a Out of 800 families with 5 children each , how many would you expect to have (1) 3 boys & 2 girls (2) 5 girls (3) 5 boys (6)

b In an experiment on immunization of cattle from TB, the following results were obtained. (6)

	Affected	Not Affected	Total
Inoculated	267	27	294
Not Inoculated	757	155	912
Total	1024	182	1206

Use Chi-square test to determine the efficacy of vaccine in preventing TB

c Using Kuhn-Tucker conditions , solve the following NLPP (8)

Maximize $Z = 10\,x_1 + 10\,x_2 - x_1^{\,2} - x_2^{\,2}$

Subject to $x_1 + x_2 \le 8$

$-x_1 + x_2 \le 5$

Q.6.a Using Green's theorem, evaluate $\oint_c (e^{x^2} - xy)\,dx - (y^2 - ax)\,dy$ where c is the circle $x^2 + y^2 = a^2$ (6)

b Suppose that in a certain region , the daily rainfall (in inches) is a continuous random variable X with probability density function f(x) is given by (6)

$f(x) = \frac{3}{4}(2x - x^2)$, $0 \le x \le 2$

$= 0$, elsewhere

Find the probability that on a given day in this region, the rain fall is (1) not more than1 inch (2) greater than 1.5 inches (3) between 0.5 and 1.5 inches

c Find the coefficient of regression and hence the equations of the lines of regression for the following data. (8)

X	78	36	98	25	75	82	90	62	65	39
Y	84	51	91	60	68	62	86	58	53	47

Estimate the value of y when x = 50 and also estimate the value of x when y = 90

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