SE Instru. Sem-IV (R) DEC 2018

Paper / Subject Code: 41101 / Applied Mathematics-IV

Applied Maths-IV

S.E. (Sem-IV) (Instrumentation) (Choice Based)

Q.P. Code : 37609

NOV- 2018

19/11/2018

1/2

(3 Hours) Total Marks: 80

Note:- 1) Question number **1** is **compulsory**.
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the right indicates **full** marks.

Q.1 a) Verify Cauchy's Schwartz inequality for the vectors $u = (-4,2,1)$ and $v = (8,-4,-2)$ 05

b) Show that the matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is non derogatory. 05

c) Let X be a continuous variable with probability density function $f(x) = kx(1-x),\ 0 \le x \le 1$. Find k and determine a number b such that $P(X \le b) = P(X \ge b)$. 05

d) Evaluate $\int_C (\bar{z} + 2z)dz$, where C is 05

i) The upper half of the circle $|z| = 2$.

ii) The lower half of the circle $|z| = 2$.

Q.2 a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. 06

b) If $f(a) = \int_C \frac{4z^2+z+5}{z-a} dz$, where C is $|z| = 2$, find the values of $f(1), f(i)\ f'(-1), f''(-i)$ 06

c) A random variable X has the following Probability function 08

X	:	1	2	3	4	5	6	7
$P(X=x)$	:	k	$2k$	$3k$	k^2	k^2+k	$2k^2$	$4k^2$

Find i) k ii) $P(X<5)$ iii) $P(X>5)$ iv) $P\left(\frac{X<5}{2<X\le 6}\right)$

Q.3 a) The equation of the two regression lines are $3x + 2y = 26$ and $6x + y = 31$. Find i) mean of x and y ii) coefficient of correlation between x and y iii) σ_y if $\sigma_x = 3$ 06

b) Fit a Binomial distribution to the following data 06

X:	0	1	2	3	4	5	6
Y:	5	18	28	12	7	6	4

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c) Examine whether the set of real numbers with operations of addition and multiplication defined as $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$; $k(x_1, y_1) = (3kx_1, 3ky_1)$ is a vector space — 08

Q.4 a) Construct an orthonormal basis of R^3 using Gram-Schmidt process to $S = \{(3,0,4), (-1,0,7), (2,9,1)\}$ — 06

b) A continuous random variable has probability density function $f(x) = kx^2(1 - x^3)$ $0 < x < 1$. Find i) k ii) mean iii) variance — 06

c) Show that the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ is diagonalizable. Also find the transforming matrix and diagonal matrix. — 08

Q.5 a) Verify Caley-Hamilton theorem for $A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ Hence find A^{-1} — 06

b) Find Karl Pearson's coefficient of correlation and also, the spearman's rank coefficient of correlation for the following data. — 06

X:	12	17	22	27	32
Y:	113	119	117	115	121

c) Obtain Taylor's and Laurent's series for $f(z) = \frac{z^2-1}{z^2+5z+6}$ around z=0. — 08

Q.6 a) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$, using Cauchy's residue theorem — 06

b) A random variable X has the following probability density function $f(x) = \begin{cases} ke^{-kx}, & x > 0, k > 0 \\ 0, & elsewhere \end{cases}$ — 06

Find m.g.f. and hence find mean and variance

c) For the normal variate with mean 2.5 and standard derivation 3.5, find the probability that i) $2 \le X \le 4.5$ ii) $-1.5 \le X \le 5.3$ — 08

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