Q.P. Code : 13607

(3 Hours) **[Total marks : 80**

Note : -
1) Question number **1** is **compulsory.**
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicate **full** marks.

Q.1 a) Find the Laplace transform of $\cos t \cos 2t \cos 3t$. 05

b) Construct an analytic function whose real part is $e^x \cos y$. 05

c) Find the directional derivative of $\emptyset = x^4 + y^4 + z^4$ at point $A\ (1, -2, 1)$ in the direction of AB where B is $(2, 6, -1)$. 05

d) Expand $f(x) = lx - x^2,\ 0 < x < l$ in a half-range sine-series. 05

Q.2 a) Find the angle between the normals to the surface $xy = z^2$ at the points $(1, 4, 2), (-3, -3, 3)$. 06

b) Find the Fourier series for 06

$$f(x) = \begin{cases} -c & -a < x < 0 \\ c, & 0 < x < a \end{cases}$$

c) Find the inverse Laplace transform of 08

(i) $\dfrac{4s + 12}{s^2 + 8s + 12}$

(ii) $\log\left(\dfrac{s^2 + a^2}{\sqrt{s + b}}\right)$

Q. 3 a) State true or false with proper justification "There does not exists an analytic function whose real part is $x^3 - 3x^2y - y^3$. 06

b) Prove that $J_{5/2}(x) = \sqrt{\dfrac{2}{\pi x}}\left(\dfrac{3 - x^2}{x^2}\sin x - \dfrac{3}{x}\cos x\right)$. 06

c) Expand $f(x) = 4 - x^2$ in the interval (0, 2). 08

Q. 4 a) Use Gauss's Divergence theorem to evaluate $\iint_S \bar{N} \cdot \bar{F}\ dS$ where $\bar{F} = 4x\,i + 3y\,j - 2z\,k$ and S is the surface bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. 06

TURN OVER

b) Prove that 06
$\int x^3 \cdot J_0(x)\, dx = x^3 \cdot J_1(x) - 2x^2 \cdot J_2(x)$.

c) Solve using Laplace transform $\frac{dy}{dt} + 3y = 2 + e^{-t}$ with $y(0) = 1$. 08

Q. 5 a) Find Laplace transform of $(1 + 2t - 3t^2 + 4t^3)H(t-2)$ where 06

$$H(t-2) = \begin{cases} 0, & t < 2 \\ 1, & t \geq 2 \end{cases}$$

b) Prove that $2\, J_0''(x) = J_2(x) - J_0(x)$. 06

c) Obtain complex form of Fourier Series for $f(x) = e^{ax}$ in $(-\pi, \pi)$ where a is not an integer. Hence deduce that when α is a constant other than an integer 08

$$\sin \alpha x = \frac{\sin \pi \alpha}{i\pi} \sum \frac{(-1)^n n}{(\alpha^2 - n^2)} e^{inx}$$

Q. 6 a) Using Green's theorem evaluate 06

$$\oint_C (e^{x^2} - xy)\, dx - (y^2 - ax)\, dy$$

where C is the circle $x^2 + y^2 = a^2$.

b) Express the function 06

$$f(x) = \begin{cases} 1 & for\ |x| < 1 \\ 0 & for\ |x| > 1 \end{cases}$$

as a Fourier Integral.

c) Under the transformation $w = (1 + i)z + (2 - i)$, find the region in the w -plane into which the rectangular region bounded by $x = 0, y = 0, x = 1, y = 2$ in the z -plane is mapped. 08

__________XXX_______XXX__________