Q. P. Code: 37525

(3 hours) **Total marks: 80**

N.B.: (1) Question **No. 1** is **compulsory**

(2) Attempt **any Three** from remaining

Q1 a) If X_1 has mean 4 and variance 9 & X_2 has mean -2 and variance 4 where X_1 & X_2 are independent, find $E(2X_1 + X_2 - 3)$ and $V(2X_1 + X_2 - 3)$. [5]

b) Find the extremals of $\int_{x_1}^{x_2}(x + y')y'dx$ [5]

c) Verify Cauchy Schwartz inequality for the vectors $u = (-4, 2, 1)$ and $v = (8, -4, -2)$ [5]

d) Check whether $A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ is derogatory or not. [5]

Q2. a) Using Cauchy's Residue theorem evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}$ where C is $|z| = 4$ [6]

b) Show that the extremal of the isoperimetric problem $I[y(x)] = \int_{x_1}^{x_2}(y')^2\,dx$ subject to the condition $\int_{x_1}^{x_2} y\,dx = k$ is a parabola. [6]

c) Is the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ diagonalisable? If so find the diagonal matrix and the transforming matrix. [8]

Q3. a) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ hence find A^{-1} [6]

b) Check whether the following are subspaces of $\mathbb{R}^3$ [6]

(i) $W = \{(a, 0,0) \mid a \in \mathbb{R}\}$

(ii) $W = \{(x, y, z) \mid x = 1, z = 1, y \in \mathbb{R}\}$

c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in Taylors & Laurent's series indicating regions of convergence. [8]

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Q4 a) Using Rayleigh-Ritz method to solve the boundary value problem [6]

$I = \int_0^1 (2xy + y^2 - (y')^2)\ dx$; $0 \le x \le 1$ given $y(0) = y(1) = 0$

b) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ then prove that $3 \tan A = A \tan 3$. [6]

c) If sizes of 10,000 items are normally distributed with mean 20 cms & standard deviation of 4 cms. Find the probability that an item selected at random will have size : [8]

(i) between 18 cms and 23 cms , (ii) above 26 cms

Q5 a) Find orthonormal basis of $\mathbb{R}^3$ using Gram-Schmidt process where [6]

$S = \{(1,0,0)\ ,\ (3,7,-2)\ ,\ (0,4,1)\}$

b) In a factory, machines A , B & C produce 30%, 50% & 20% of the total production of an item. Out of their production 80% , 50% & 10% are defective respectively. An item is chosen at random and found to be defective. What is the probability that it was produced by machine A. [6]

c) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$ [8]

Q6 a) Evaluate $\int_C \frac{dz}{z^3(z+4)}$ where C is the circle [6]

(i) $|z| = 2$ and (ii) $|z - 3| = 2$

b) Two unbiased dice are thrown three times, using Binomial distribution find the probability that the sum nine would be obtained (i) once , (ii) twice [6]

c) For the following data [8]

X	100	110	120	130	140	150	160	170	180	190
Y	45	51	54	61	66	70	74	78	85	89

Find the coefficients of regression b_{xy} & b_{yx} and the coefficient of correlation (r)

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