

(3 Hours)

[Total Marks: 80]

- N.B.: 1) Question No. 1 is Compulsory.  
 2) Answer any THREE questions from Q.2 to Q.6.  
 3) Figures to the right indicate full marks.

1) a) If  $A = \begin{bmatrix} 2 & 4 \\ 0 & 3 \end{bmatrix}$  then find the eigen values of  $6A^{-1} + A^3 + 2I$  [05]

b) Determine whether the given vectors  $u = (-4, 6, -10, 1), v = (2, 1, -2, 9)$  are orthogonal with respect to the Euclidean inner product [05]

c) The probability density function of a random variable  $x$  is zero except at  $x = 0, 1, 2$  and  $p(0) = 3\alpha^3, p(1) = 4\alpha - 10\alpha^2, p(2) = 5\alpha - 1$ . Find  $\alpha$  [05]

d) Evaluate  $\oint_c \frac{z+6}{z^2-4} dz$  where  $c$  is (i)  $|z|=1$  (ii)  $|z-2|=1$ . [05]

2) a) Using Rayleigh-Ritz method, find an appropriate solution for the extremal of the functional  $I = \int_0^1 [2xy - y^2 - y'^2] dx$  given  $y(0)=y(1)=0$  [06]

b) Using Cauchy's Residue theorem evaluate  $\int_0^{2\pi} \frac{d\theta}{5 + 4 \cos \theta}$  [06]

c) A random variable  $X$  has the probability distribution given below:

$X=x$	-2	3	1
$P(X=x)$	1/3	1/2	1/6

Find i) the moment generating function ii) the first four moments about the origin [08]

3) a) Compute  $A^9 - 6A^8 + 10A^7 - 3A^6 + A + I$  where  $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$  [06]

b) Verify Cauchy-Schwartz inequality for the vectors  $u = (-4, 2, 1)$  &  $v = (8, -4, -2)$  [06]

c) Obtain Taylor's or Laurent's series expansion of the function  $f(z) = \frac{1}{z^2 - 3z + 2}$  when (i)  $|z| < 1$  (ii)  $1 < |z| < 2$  [08]

- 4) a) Obtain the equation of the line of regression of Y on X for the following data and estimate Y when X = 73 [06]

X	70	72	74	76	78	80
y	163	170	179	188	196	200

- b) Show that the functional  $\int_{x_1}^{x_2} [y^2 + x^2 y'] dx$  assumes extreme values on the straight line  $y = x$  [06]

c) Let  $R^3$  have the Euclidean inner product. Use the Gram-Schmidt process to transform

the basis vectors  $u_1=(1,0,0), u_2=(3,7,-2), u_3=(0,4,1)$  into an orthonormal basis [08]

- 5) a) Evaluate  $\int_c \frac{1}{z} \cos z dz$  where c is the ellipse  $9x^2 + 4y^2 = 1$  [06]

b) Seven dice are thrown 729 times. How many times do you expect at least four 10 dice to show three or five? [06]

- c) Show that the matrix  $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$  is diagonalisable. Find the diagonal form D and the diagonalising matrix M. [08]

- 6) a) A continuous random variable X has the p.d.f. defined by  $f(x) = A + Bx, 0 \leq x \leq 1$ . If the mean of the distribution is  $\frac{1}{3}$  find A and B [06]

- b) Find  $e^A$ , if  $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \\ 1 & 3 \\ 2 & 2 \end{bmatrix}$  [06]

- c) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$  ( $a > 0, b > 0$ ) [08]

\*\*\*\*\*