S.E. Sem III (CBGS) (ETRX & EXTC). 20/5/15

Applied Maths-III

QP Code : **4787**

(3 Hours) [Total Marks : 80

[Revised Course]

N.B.: 1) Question No.1 is compulsory.

2) Attempt any three from the remaining questions.

3) Assume suitable data if necessary.

1. (a) Determine the constants a,b,c,d if $f(z) = x^2 + 2axy + by^2 + i(dx^2 + 2cxy + y^2)$ is analytic. 5

(b) Find a cosine series of period 2π to represent $\sin x$ in $0 \le x \le \pi$ 5

(c) Evaluate by using Laplace Transformation $\int_0^{\infty} e^{-3x}\, t \cos t\, dt.$ 5

(d) A vector field is given by $\bar{F} = (x^2+xy^2)\, i + (y^2 + x^2 y)\, j$. Show that $\bar{F}$ is irrotational and find its scalar potential. Such that $\bar{F} = \nabla \phi$. 5

2. (a) Solve by using Laplace Transform 6

$(D^2 + 2D + 5)\, y = e^{-t} \sin t$, when $y(0) = 0$, $y'(0) = 1$.

(b) Find the total work done in moving a particle in the force field 6

$\bar{F} = 3xy\, i - 5z\, j + 10x\, k$ along $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from t=1 and t=2.

(c) Find the Fourier series of the function $f(x) = e^{-x}$, $0 < x < 2\pi$ and 8

$f(x + 2\pi) = f(x)$. Hence deduce that the value of $\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2+1}$.

3 (a) Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} . \sin x$ 6

(b) Verify Green's theorem in the plane for $\oint (x^2 - y)\, dx + (2y^2 + x)\, dy$ 6

Around the boundary of region defined by $y = x^2$ and $y = 4$.

(c) Find the Laplace transforms of the following. 8

i) $e^{-t} \int_0^t \frac{\sin u}{u}\, du$ ii) $t\sqrt{1 + \sin t}$

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4 (a) If $f(x) = C_1Q_1(x) + C_2Q_2(x) + C_3Q_3(x)$, where C_1, C_2, C_3 constants and Q_1, Q_2, Q_3 are orthonormal sets on (a,b), show that 6

$\int_a^b [f(x)]^2\,dx = c_1^2 + c_2^2 + c_3^2$.

(b) If $v = e^x \sin y$, prove that v is a Harmonic function. Also find the corresponding harmonic conjugate function and analytic function. 6

(c) Find inverse Laplace transforms of the following. 8

i) $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ ii) $\frac{s+2}{s^2-4s+13}$

5 (a) Find the Fourier series if $f(x) = |x|$, $-k < x < k$ 6

Hence deduce that $\sum \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$.

(b) Define solenoidal vector. Hence prove that $\bar{F} = \frac{\bar{a} \times \bar{r}}{r^n}$ is a solenoidal vector 6

(c) Find the bilinear transformation under which 1, i, -1 from the z-plane are mapped onto 0, 1, ∞ of w-plane .Further show that under this transformation the unit circle in w-plane is mapped onto a straight line in the z-plane .Write the name of this line. 8

6 (a) Using Gauss's Divergence Theorem evaluate $\iint_s \bar{F} . d\bar{s}$ where $\bar{F} = 2x^2yi - y^2j + 4xz^2k$ and s is the region bounded by $y^2 + z^2 = 9$ and $x = 2$ in the first octant . 6

(b) Define bilinear transformation. And prove that in a general, a bilinear transformation maps a circle into a circle. 6

(c) Prove that $\int x J_{2/3}(x^{3/2})\,dx = -\frac{2}{3} x^{-1/2} J_{-1/3}(x^{3/2})$. 8
