[Time: Three Hours]   **[Marks:80]**

Please check whether you have got the right question paper.

N.B: 1.   Question.No.1 is compulsory.
         2.   Attempt any three from the remaining.

Q.1. a) Find the extremal of $\int_{x_0}^{x_1} \frac{1 + y^2}{y'^2} dx$ (5)

b) Is $(6,7,-4)$ a linear combination of $v_1 = (1,2,2)$, $v_2 = (3,4,6)$ (5)

c) Check whether $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ is derogatory or not.   (5)

d) Evaluate $\int_0^{1+i} z^2 dz$, along the parabola $x = y^2$ (5)

Q.2. a) Show that the functional $\int_0^{\pi/2} \left\{ 2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \right\} dt$; such that $x(0) = 0$, $x\left(\frac{\pi}{2}\right) = -1$,

$y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 1$ is stationary if x=-sint, y=sint.   (6)

b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx$, $a > 0, b > 0$ (6)

c) Reduce the quadratic form $x^2 - 2y^2 + 10z^2 - 10xy + 4xz - 2zy$ to canonical form and
hence, find its rank, index and signature and value class.   (8)

Q.3. a) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1} & A^4 (6)

b) Using Residue theorem evaluate $\int_C \frac{e^z}{z^2 + \pi^2} dz$ where C is $|z|=4$.   (6)

c) Find the singular value decomposition of $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$ (8)

Q.4. a) If $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$, prove that 3tanA=Atan3   (6)

b) Find the sum of the residues at singular points of $f(z) = \frac{z - 4}{z(z - 1)(z - 2)}$ (6)

c) Check whether the set of real numbers (x,0) with operation $(x_1,0) + (x_2,0) = (x_1 + x_2,0)$, and $k(x_1,0) = (kx_1,0)$ is a vector space. **(8)**

Q.5. a) Find the extremum of $\int_{x_0}^{x_1} (2xy - y''^2) dx$. **(6)**

b) Construct an orthonormal basis of R^3 using Gram Schmidt process to
   S = {(3,0,4),(-1,0,7),(2,9,11)} **(6)**

c) Find all possible Laurent's expansions of $\dfrac{2z-3}{z^2 - 4z - 3}$ about $z = 4$. **(8)**

Q.6. a) Find the linear transformation Y=AX which carries $X_1 = (1,1,-1)'$, $X_2 = (1,-1,1)'$, $X_3 = (-1,1,1)'$
   onto $Y_1 = (2,1,3)'$, $Y_2 = (2,3,1)'$, $Y_3 = (4,1,3)'$ **(6)**

b) Show that the vectors $v_1 = (1,2,4)$, $v_2 = (2,-1,3)$, $v_3 = (0,1,2)$ are linearly independent .
   Express $v_4 = (-3,7,2)$ in terms of v_1, v_2, v_3 **(6)**

c) If C is circle |z|=1, using the integral $\int_C \dfrac{e^{kz}}{z} dz$ where k is real, show that

$$\int_0^\pi e^{k \cos \theta} \cos(k \sin \theta) d\theta = \pi$$ **(8)**