DURATION : 3 HOURS **MAX.MARKS:80**

1) Question No.1 is compulsory
2) Attempt any THREE of the remaining
3) Figures to the right indicate full marks.

Q1

A) Find Laplace transform of $f(t) = \sin^5 t$ 5

B) Prove that $u = x^2 - y^2$ is harmonic function also find corresponding analytic function f(z) 5

C) Find the half range sine series of $f(x) = 2x$ in $(0, \pi)$ 5

D) Find the Unit normal vector to the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2,-1,2) hence find angle between them 5

Q2

A) Prove that $J_{(-3/2)}(x) = -\sqrt{\frac{2}{\pi x}}\,.\left(\frac{\cos x}{x} + \sin x\right)$ 6

B) Find the Bilinear transformation which maps the points z = 1 , i , -1 onto the points w = 0 , 1, ∞ 6

C) Obtain the fourier series for $f(x) = x\cos x$ in $(-\pi, \pi)$ 8

Q3

A) Find inverse laplace transform of 6

(i) $\log\left(\frac{1+s^2}{4+s^2}\right)$ (ii) $\frac{s+5}{(s+4)^3}$

B) Show that the of functions {cosx ,cos3x , cos5x ,................} is an orthogonal over [0 , π/2] .Hence construct orthonormal set of functions. 6

C) Prove that $y = \sqrt{x}\,.J_n(x)$ is a solution of the equation , 8

$x^2 \frac{d^2y}{dx^2} + (x^2 - n^2 + \frac{1}{4})y = 0$

Q4

A) Prove that $\int x^4 J_1(x)dx = x^4 J_2(x) - 2x^3 J_3(x)$ 6

B) Use Gauss's Divergence theorem to evaluate $\iint_S \bar{N}.\bar{F}ds$ where $\bar{F}$=4xi +3yj -2zk and S is the surface bounded by x=0 ,y=0 ,z= 0 and 2x +2y +z=4 6

C) Solve using Laplace transform(D^2 +2D+1)y =3te^{-t} ,given y(0)=4 and y'(0)=2 8

Q5

A Find Fourier series for 6

$$f(x)= \begin{cases} \pi + x , & 0 < x < \pi \\ \pi - x , & -\pi < x < 0 \end{cases}$$

B) Find the image of the region bounded by x+y=0 , x=y , x+y=1 ,x-y=1 under the bilinear transformation w = 2z +2i 6

c) Prove that $\bar{F}$=(y^2cosx + z^3)i+(2ysinx-4)j+(3xz^2+2)k is a conservative field .Find (i)Scalar Potential for $\bar{F}$ (ii) The work done in moving an object in this field from (0 ,1,-1) to($\frac{\pi}{2}$, -1 ,2). 8

Q6

A) Find the Laplace Transform of $e^{-t}\int_0^t sin3ucos2udu$ 6

B) Find Complex form of Fourier Series of sinh2x in (-2 , 2) 6

C) Express the function 8

$$f(x) = \begin{cases} 1 & , \quad |x| < 1 \\ 0 & , \quad |x| > 1 \end{cases}$$ as Fourier integral .Hence evaluate

$$\int_0^\infty \frac{sinw.coswx}{w} dw$$

****