SE - sem - IV - CBGS - EE / EXTC 30/11/15

AM - IV

QP Code : 5350

Duration: 3 Hours (REVISED COURSE) Total marks assigned to the paper:80

N.B:1) Q 1 is compulsory.

2) Attempt any three from the remaining.

Q 1: a) Find the extremal of $\int_{x_1}^{x_2}(y^2 - y'^2 - 2y\cosh x)dx$ (5)

b) Find an orthonormal basis for the subspaces of R^3 by applying Gram-Schmidt process where $S=\{(1,2,0)(0,3\ 1)\}$ (5)

c) Show that eigen values of unitary matrix are of unit modulus. (5)

d) Evaluate $\int \frac{dz}{z^3(z+4)}$ where $|z| = 4$. (5)

Q2: a) Find the complete solution of $\int_{x_0}^{x_1}(2xy - y''^2)\,dx$ (6)

(b) Find the Eigen value and Eigen vectors of the matrix A^3 where $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ (6)

(c) Find expansion of $f(z) = \frac{1}{(1+z^2)(z+2)}$ indicating region of convergence. (8)

Q3: a) Verify Cayley Hamilton Theorem and find the value of A^{64} for the matrix A=$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. (6)

b) Using Cauchy's Residue Theorem evaluate $\int_{-\infty}^{\infty} \frac{x^2}{x^6+1}dx$ (6)

c) Show that a closed curve 'C' of given fixed length (perimeter) which encloses maximum area is a circle. (8)

Q4: a) State and prove Cauchy-Schwartz inequality. Verify the inequality for vectors $u = (-4,2,1)$ *and* $v = (8,-4,-2)$ (6)

b) Reduce the Quadratic form $xy + yz + zx$ to diagonal form through congruent transformation.(6)

c) If $A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$ then find e^A *and* 4^A with the help of Modal matrix. (8)

Q5: a) Solve the boundary value problem $\int_0^1(2xy + y^2 - y'^2)\,dx$, $0 \le x \le 1,\ y(0) = 0, y(1) = 0$ by Rayleigh - Ritz Method. (6)

b) If $W = \{\alpha : \alpha \in R^n \text{ and } a_1 \geq 0\}$ a subset of $V = R^n$ with $\alpha = (a_1, a_2 \ldots \ldots \ldots a_n)$ in R^n (n≥3). Show that W is not a subspace of V by giving suitable counter example. (6)

c) Show that the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ is similar to diagonal matrix. Find the diagonalsing matrix and diagonal form. (8)

Q6: a) State and prove Cauchy's Integral Formula for the simply connected region and hence evaluate

$$\int \frac{z+6}{z^2-4} dz, \quad |z-2| = 5$$ (6)

b) Show that $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta} d\theta = \frac{2\pi}{b^2}\left(a - \sqrt{a^2 - b^2}\right), \; 0 < b < a.$ (6)

c) Find the Singular value decomposition of the following matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$ (8)

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