(Revised course)

Time: 3 hours

Total marks:80

N.B: (1) Question No.1 is compulsory.

- (2) Answer any three questions from remaining.
- (3) Assume suitable data if necessary.



I. (a)

$$\int_{0}^{\infty} e^{-t} \left(\frac{\cos 3t - \cos 2t}{t} \right) dt$$

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(b) Obtain the Fourier Series expression for f(x) = 2x - 1 in (0,3)

05

(c) Find the value of 'p' such that the function

$$f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\left(\frac{py}{x}\right)$$
 is analytic.

05

(d) If $\overline{F} = (y \sin z - \sin x)\hat{i} + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$. Show that \overline{F} is irrotational .Also find its scalar potential.

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2. (a) Solve the differential equation using Laplace Transform

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$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-y}$$
, given y(0)=4 and y'(0)=2

(b) Prove that

$$J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) - \left(\frac{24}{x^2} - 1\right)J_0(x)$$

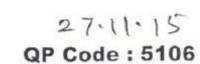
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(c) i) In what direction is the directional derivative of $\phi = x^2y^2z^4$ at (3,-1,-2) maximum. Find its magnitude.

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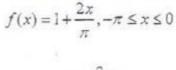
ii) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Prove that $\nabla r'' = nr^{n-2}r$





3. (a) Obtain the Fourier Series expansion for the function



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$$=1-\frac{2x}{\pi}, 0 \le x \le \pi$$

(b) Find an analytic function f(z) =u+iv where.





$$u - v = \frac{x - y}{x^2 + 4xy + y^2}$$

(c) Find Laplace transform of



- i) $\cosh t e^u \sinh u$
- ii) $t\sqrt{1+\sin t}$



4. (a) Obtain the complex form of Fourier series for $f(x) = e^{\alpha x}$ in (-L,L)



(b) Prove that $\int x^4 J_1(x) dx = x^4 J_1(x) - 2x^3 J_2(x) + \varepsilon$

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(c) Find

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ii)
$$L^{-1}\left[\cot^{-1}\left(\frac{s+3}{2}\right)\right]$$

- 5. (a) Find the Bi-linear Transformation which maps the points 06 1,i,-1 of z plane onto 0,1,∞ of w-plane

(b) Using Convolution theorem find

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$$L^{-1} \left[\frac{s^2}{\left(s^2 + 4\right)^2} \right]$$

- (c) Verify Green's Theorem for $\int_{C} \overline{F} \cdot dr$ where $\overline{F} = (x^2 y^2)\hat{i} + (x + y)\hat{j}$ and C is the triangle with vertices (0,0), (1,1) and (2,1)
- 6. (a) Obtain half range sine series for $f(x) = x, 0 \le x \le 2$

 $=4-x, 2 \le x \le 4$

- (b) Prove that the transformation $w = \frac{1}{z+i}$ transforms the real axis of the z-plane into a circle in the w-plane.
- (c) i) Use Stoke's Theorem to evaluate $\int_{c}^{\infty} \overline{F} \cdot dr$ where $\overline{F} = (x^{2} y^{2})\hat{i} + 2xy\hat{j}$ and C is the rectangle in the plane z=0, bounded by x=0, y=0, x=a and y=b.
 - ii) Use Gauss Divergence Theorem to evaluate $\iint_{S} \overline{F} \cdot \hat{h} ds \text{ where } \overline{F} = 4x\hat{i} + 3y\hat{j} 2z\hat{k} \text{ and S is the surface bounded by } x=0, y=0, z=0 \text{ and } 2x+2y+z=4$