S.E. (1) Electrical - III
(2) Electronics - III CBGS
(3) EXTC - III

26.11

(3 Hours)

QP Code : 14571

[Total Marks : 80

N.B. (1) Question No.1 is compulsory.
(2) Attempt any three questions out of the remaining five questions.
(3) Figures to right indicate full marks.

1. (a) Prove that $f(z)=x^2-y^2+2ixy$ is analytic and find $f'(z)$ 5
(b) Find the Fourier series expansion for $f(x)=|x|, \ in \ (-\pi,\pi)$ 5
(c) Using laplace transform solve the following differential equation with given condition $\frac{d^2y}{dt^2}+y=t, \ given\ that\ y(0)=1\ \&\ y'(0)=0$ 5
(d) If $\overline{A}=\nabla(xy+yz+zx), \ find\ \nabla\cdot\overline{A}\ and\ \nabla\times\overline{A}$ 5

2. (a) If $L[J_0(t)]=\frac{1}{\sqrt{s^2+1}}$, prove that $\int_0^\infty e^{-6t}t\,J_0(4t)\,dt=3/500$ 6
(b) Find the directional derivative of $\phi=x^4+y^4+z^4\ at\ A(1,-2,1)$ in the direction of AB where $B\ is\ (2,6,-1)$. Also find the maximum directional derivative of ϕ at $(1,-2,1)$. 6
(c) Find the Fourier series expansion for $f(x)=4-x^2, \ in\ (0,2)$
Hence deduce that $\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}......$ 8

3. (a) Prove that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ 6
(b) Using Green's theorem evaluate $\int_C(2x^2-y^2)dx+(x^2+y^2)dy$ where 'c' is the boundary of the surface enclosed by the lines $x=0,\ y=0,\ x=2,\ y=2$. 6
(c) i) Find Laplace Transform of $e^{-3t}\int_0^t u\sin 3u\,du$
ii) Find the Laplace transform of $\frac{d}{dt}\left(\frac{1-\cos 2t}{t}\right)$ 8

4. (a) Obtain complex form of Fourier series for the functions $f(x)=\sin ax \ \ in\ \ (-\pi,\pi)$, where a is not an integer. 6
(b) Find the analytic function whose imaginary part is $v=\frac{x}{x^2+y^2}+\cosh y\cdot\cos x$ 6
(c) Find inverse Laplace Transform of following
i) $\log\left(\frac{s^2+a^2}{\sqrt{s+b}}\right)$ ii) $\frac{1}{s^3(s-1)}$ 8

5. (a) Obtain half-range cosine series for $f(x)=x(2-x)\ in\ 0<x<2$ 6
(b) Prove that $\overline{F}=\frac{\overline{r}}{r^3}$ is both irrotational and solenoidal 6
(c) Show that the function $u=\sin x\cosh y+2\cos x\sinh y+x^2-y^2+4xy$ satisfies

Laplace's equation and find it corresponding analytic function 8

6. (a) Evaluate by Stoke's theorem $\int_C (x\,y\,dx + x\,y^2\,dy)$ where C is the square in the xy-plane with vertices $(1,0),(0,1),(-1,0), and\ (0,-1)$ 6

(b) Find the bilinear transformation, which maps the points $z=-1,1,\infty$ onto the points $w=-i,-1,i$. 6

(c) Show that the general solution of $\dfrac{d^2y}{dx^2}+4x^2y=0$ is $y=\sqrt{x}\left[A\,J_{1/4}(x^2)+B\,J_{-1/4}(x^2)\right]$ where A and B are constants. 8
