Duration – 3 Hours **Total Marks : 80**

N.B.:- 1. **Question no 1 is compulsory.**
2. **Attempt any THREE questions out of remaining FIVE questions.**

Q.1 a) Find Laplace Transform of the given function $f(t) = e^{-5t}\, erf\left(\sqrt{t}\right)$. **(5)**

b) Prove that $J_{-\frac{3}{2}} = -\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x} + \sin x\right)$ **(5)**

c) Find complex form of Fourier series of $cosh 2x + sinh 2x$; $(-\pi, \pi)$. **(5)**

d) Find the Directional derivative of $F = x^2 - y^2 + 2z^2$ at P(1,2,3) in the direction of the line PQ where Q is the point (5,0,4).In what direction will it be maximum? What is the magnitude of this maximum? **(5)**

Q.2 a) Prove that $\nabla \times \left[\frac{\bar{a} \times \bar{r}}{r^3}\right] = \frac{-\bar{a}}{r^3} + \frac{3(\bar{a} \bullet \bar{r})\,\bar{r}}{r^5}$. **(6)**

b) Show that the set of functions $\left\{Sin\left(\frac{\pi x}{2L}\right), Sin\left(\frac{3\pi x}{2L}\right), Sin\left(\frac{5\pi x}{2L}\right), ...\right\}$ forms an orthogonal set over the interval $[0, L]$. Construct corresponding orthonormal set. **(6)**

c) Determine the analytic function f(z) = u + iv if 3u + 2v = $y^2 - x^2 + 16xy$. **(8)**

Q.3 a) Find the Bilinear transformation that maps the points z = 1, i, -1 into w = 0,1,∞. **(6)**

b) Prove that $\int_0^\infty \frac{e^{-\sqrt{2}t} \sin t \sinh t}{t}\, dt = \frac{\pi}{8}$ **(6)**

c) Obtain Fourier series of $f(x) = \begin{cases} x + \pi/2 & -\pi \le x \le 0 \\ \pi/2 - x & 0 \le x \le \pi \end{cases}$ **(8)**

Hence deduce that $\frac{\pi^4}{96} = \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots$

Q.4 a) Find the Fourier sine transform of e^{-x}, x≥ 0 ,and hence deduce that $\int_0^\infty \frac{x \sin mx}{1+x^2}\, dx = \frac{\pi}{2} e^{-m}, m \geq 0$. **(6)**

b) Find Inverse Laplace Transform of $\frac{(s+2)^2}{(s^2+4s+8)^2}$ using Convolution theorem. **(6)**

c) Verify Green's theorem for $\bar{F} = (x^2 - xy)i + (x^2 - y^2)\,j$ and C is the closed curve formed by $x^2 = 2y, x = y$. **(8)**

Q.5 a) Prove that $\int J_5(x)dx = -J_4 - \frac{4}{x}J_3(x) - \frac{8}{x^2}J_2(x)$. **(6)**

b) Evaluate $\iint_S \overline{F} \bullet \overline{n} dS$ where S is the surface of the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$ and $\overline{F} = 4xi - 2y^2 j + z^2 k$. **(6)**

c) Find inverse Laplace transform of

(i) $\log\left(\frac{s^2 + a^2}{(s+b)^2}\right)$ **(4)**

(ii) $\frac{e^{-2s}}{s^2 + 8s + 25}$ **(4)**

Q.6 a) Prove that $\overline{F} = (6xy^2 - 2z^3)i + (6x^2y + 2yz)j + (y^2 - 6z^2x)k$ is a conservative field. Find the scalar potential for $\overline{F}$. Hence find the work done in moving a particle from (1,0,2) to (0,1,1) **(6)**

b) Express the function $f(x) = -e^{-kx}$, for x<0 & $f(x) = e^{-kx}$, for $x > 0$ as Fourier integral and hence deduce that **(6)**

$\int_0^\infty \frac{\omega \sin \omega x}{\omega^2 + k^2} d\omega = \frac{\pi}{2} e^{-kx}, if\ x > 0, k > 0.$

c) Solve $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 3te^{-t}$, $y(0) = 4$, $y'(0) = 2$ using Laplace Transform. **(8)**
