Total Marks:80
Duration: 3Hrs

NB 1. Question No.1 is compulsory

2. Attempt any three from the remaining six questions

3. Figures to the right indicate full marks

Q1a If the Laplace Transform of $e^{-t}\int_0^t u\cos 2u\,du$ [20]

b Prove that $f(z)=\sinh z$ is analytic and find its derivative

c Obtain Half range Sine Series for $f(x)=x+1$ **in** $(0,\pi)$

d Find a unit vector normal to the surface $x^2y+2xz=4$ at $(2,-2,3)$

Q2 a Prove that $\overline{F}=(2xy^2+yz)i+(2x^2y+xz+2yz^2)j-(2y^2z+xy)k$ is Irrotational.

Find Scalar Potential for $\overline{F}$ **[6]**

b Find the inverse Laplace Transform using Convolution theorem

$$\frac{(s-1)^2}{(s^2-2s+5)^2}$$ [6]

c. Find Fourier Series of $f(x)=\begin{cases}\pi x; 0\le x\le 1\\ \pi(2-x); 1\le x\le 2\end{cases}$ **[8]**

Q3 a Find the Analytic function $f(z)=u+iv$ if $v=\dfrac{x}{x^2+y^2}+\cosh x\cos y$ [6]

b Find Inverse Z transform of $\dfrac{(3z^2-18z+26)}{(z-2)(z-3)(z-4)}$, $3<|z|<4$ [6]

c Solve the Differential Equation $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dx}+2y=5\sin t$, $y(0)=0$, $y'(0)=0$ using Laplace Transform [8]

Q4 a Find the Orthogonal Trajectory of $3x^2y-y^3=k$ [6]

b Find the Z-transform of $2^{\mathbf{K}}\sinh 3\mathbf{K}, \mathbf{K}\ge 0$ [6]

c Express the function $f(x)=\begin{cases}1 & ; |x|<1\\ 0 & ; |x|>1\end{cases}$ as Fourier Integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda}{\lambda}.\cos(\lambda x)\,d\lambda$ **[8]**

Q5 a Evaluate using Stoke 's theorem $\int_c (2x-y)dx - yz^2dy - y^2zdz$ where C is the circle $x^2+y^2=1$

corresponding to the sphere $x^2+y^2+z^2=1$ above the XY plane [6]

b Show that $w=\frac{2z+3}{z-4}$ maps the circle $x^2+y^2-4x=0$ into straight line $4u+3=0$ [6]

c Find Inverse Laplace Transform $i)\ e^{-s}\tanh^{-1}s$ ii) $\frac{6}{(2s+1)^3}$ [8]

Q6 a Find the Laplace transform of $f(t)=\frac{2t}{3}, 0\le t\le 3, f(t+3)=f(t)$ [6]

b Find Complex Form of Fourier Series for $\sin(\alpha x)$ ; $(-\pi,\pi)$ , $\alpha$ is not an integer [6]

c Verify Green's theorm for $\int_c (2x^2-y^2)dx+(x^2+y^2)dy$ where C is the boundary of the surface enclosed by lines x=0,y=0,x=2,y=2 [8]

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