Paper / Subject Code: 50903 / Discrete Structures

(3 hrs)   **Max. Marks: 80**

1) Question **no.1** is **compulsory**.
2) Solve any **three** questions out of remaining **five** questions.
3) All questions carry equal marks as indicated by figures to the right.
4) Assume appropriate data whenever required. State all assumptions clearly.

Q.1 a) Prove using Mathematical Induction   (05M)
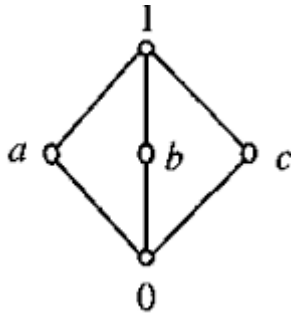
       $1^2 + 2^2 + 3^2 + ... + n^2 = n(n+1)(2n+1)/6$

b) Let A = {a,b,c}. Draw Hasse Diagram for (p(A), $\subseteq$)   (05M)

c) Let A={1,2,3,4,5}. A relation R is defined on A as aRb iff a<b. Compute R^2 and R ∞ (05M)

d) Let f : R O R, where f(x) = 2x − 1 and $f^{-1}(x) = (x+1)/2$ (05M)

    Find (f O f^{-1})(x)

Q.2 a) Define Distributive Lattice. Check if the following diagram is a Distributive lattice or not.   (04M)

b) Prove that set G = {1,2,3,4,5,6} is a finite abelian group of order 6 with respect to multiplication module 7   (08 M)

c) Find the number of positive integers not exceeding 100 that are not divisible by 5 or 7. Also draw corresponding Venn Diagram.   (08 M)

Q.3 a) Construct Truth Table and check if the following statement is tautology.

    $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$ (04 M)

b) Consider the (2,5) group encoding function defined by   (08 M)

    e(00)=00000   e(10)=10101

    e(01)=01110   e(11)=11011

  Decode the following words relative to a maximum likelihood decoding function.

  i) 11110   ii) 10011   iii) 10100

c) How many four digits can be formed out of digits 1,2,3,5,7,8,9 if no digit is repeated twice? How many of these will be greater than 3000?   (08 M)

Q.4 a) A bag contains 10 red marbles, 10 white marbles, and 10 blue marbles. What is the minimum no. of marbles you have to choose randomly from the bag to ensure that we get 4 marbles of same color? Use pigeonhole Principle.   (04 M)

b) Define Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit. Determine if following diagram has Euler Path, Euler Circuit, Hamiltonian Path and Hamiltonian Circuit and state the path /circuit. (08 M)
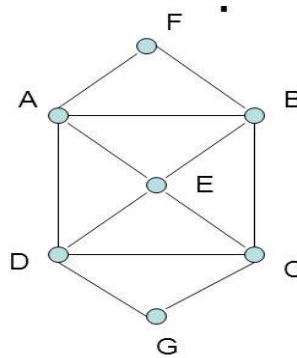
c) In how many ways a committee of three faculty members and 2 students can be formed from 7 faculty members and 8 students. (08 M)

Q.5 a) Let Z_n denote the set of integers $\{0,1,2,...n-1\}$. Let $\odot$ be a binary operation on Z_n such that a $\odot$ b= reminder of ab divided by n (04M)
  i) Construct table for the operation $\odot$ for n=4
  ii) Show that $(Z_n, \odot)$ is a semi group for any n

b) Find Transitive Closure of R represented by M_R as follows using Warshall's algorithm set {a,b,c,d}. (08M)

$$M_R= \begin{vmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

c) Let A = {1, 2, 3, 4, 5}, and let
  R = {(1, 1),(1, 3),(1, 4),(2, 2),(2, 5),(3, 1),(3, 3),(3, 4),(4, 1),(4, 3),(4, 4),(5, 2),(5, 5)}. Check if R is a equivalence relation. Justify your answer. Find equivalence classes of A. (08M)

Q.6 a) How many vertices are necessary to construct a graph with exactly 6 edges in which each vertex is of degree 2. (04 M)
  b) What is the solution of the recurrence relation $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-3}$ with $a_0=8$, $a_1=6$ and $a_2=26$? (08M)
  c) Determine if following graphs G_1 and G_2 are isomorphic or not. (08M)

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