MAY 2017

(REVISED COURSE) QP Code : 11700

(3 Hours) [Total Marks : 80]

N.B.: (1) Question No. 1 is compulsory.
(2) Answer any Three from remaining
(3) Figures to the right indicate full marks

1. (a) State Cauchy Reimann equation in polar form. Find p if $f(z) = r^2 \cos 2\theta + ir^2 \sin p\theta$ is analytic. 5
(b) Find Laplace transform of sin 2t.cos3t . 5
(c) Prove {sin n x} , n = 1,2,3........ is orthogonal w.r.t. (0, 2π) 5
(d) Evaluate $\int_{1+i}^{2+4i} (x^2 + ixy)\,dz$ along the curve $x = t$, $y = t^2$ 5

2. (a) Using Laplace transform, solve the differential equation, $\frac{dx}{dt} + 3x = 2 + e^{-t}$ with $x(0) = 1$ 6
(b) Evaluate $\oint_c \frac{z+1}{z^3 - 2z}\,dz$ where $c : |z| = 1$ 6
(c) Obtain the Taylor's and Laurent series which represent the function $\frac{z^2 - 1}{(z+3)(z+4)}$ 8
in the regions, (i) $|z| < 3$ (ii) $3 < |z| < 4$ (iii) $|z| > 4$.

3. (a) Solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$ by Bender -Schmidt method, given $u(0,t) = u(x,0) = 0$, $u(1,t) = t$ taking $h = 0.25$ 6
(b) Evaluate $\int_0^\infty te^{-3t} \sin t\,dt$ 6
(c) Obtain Half Range Sine Series of $f(x) = x(\pi - x)$ in $(0, \pi)$.Hence, evaluate $\sum_{m=0}^{\infty} \frac{(-1)^m}{(2m+1)^3}$ 8

4. (a) Find the orthogona; trajectory of the family of curves $x^3y - xy^3 = 0$ 6
(b) Find Fourier series of $f(x) = |x|$ in $(-3,3)$ 6
(c) Find the inverse Laplace transform of the following : (i) $\cot^{-1} s$ (ii) $\frac{8e^{-3s}}{s^2 + 4}$ 8

5. (a) Solve by Crank-Nichloson simplifies formula $\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$ 6
$u(0,t) = u(1,t) = 0$, $u(x,0) = 100x(1-x)$ taking $h = 0.25$ for one time step
(b) Using convolution theore, find the inverse Laplace transform of $\frac{s}{(s^2+1)(s^2+4)}$ 6
(c) Find bilinear transformation which maps the poins $z = 1, i, -1$ onto the points $w = i, 0, -1$.Hence, find the image of $|z| \le 1$ onto the w - plane 8

6. (a) Using residue theorem, evaluate $\int_0^{\infty} \frac{dx}{x^2+1}$ 6

(b) Obtain Complex form of Fourier series for $f(x) = e^{ax}$ over $-\pi < x < \pi$ 6

(c) Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = C^2 \frac{\partial^2 u}{\partial x^2}$

under boundry condition $u(0,t) = u(l,t) = 0$, $u(x,0) = x$, l being the legnth of rod. 8